# Quantifying uncertainties in the nuclear-matter equation of state

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**Christian Drischler**

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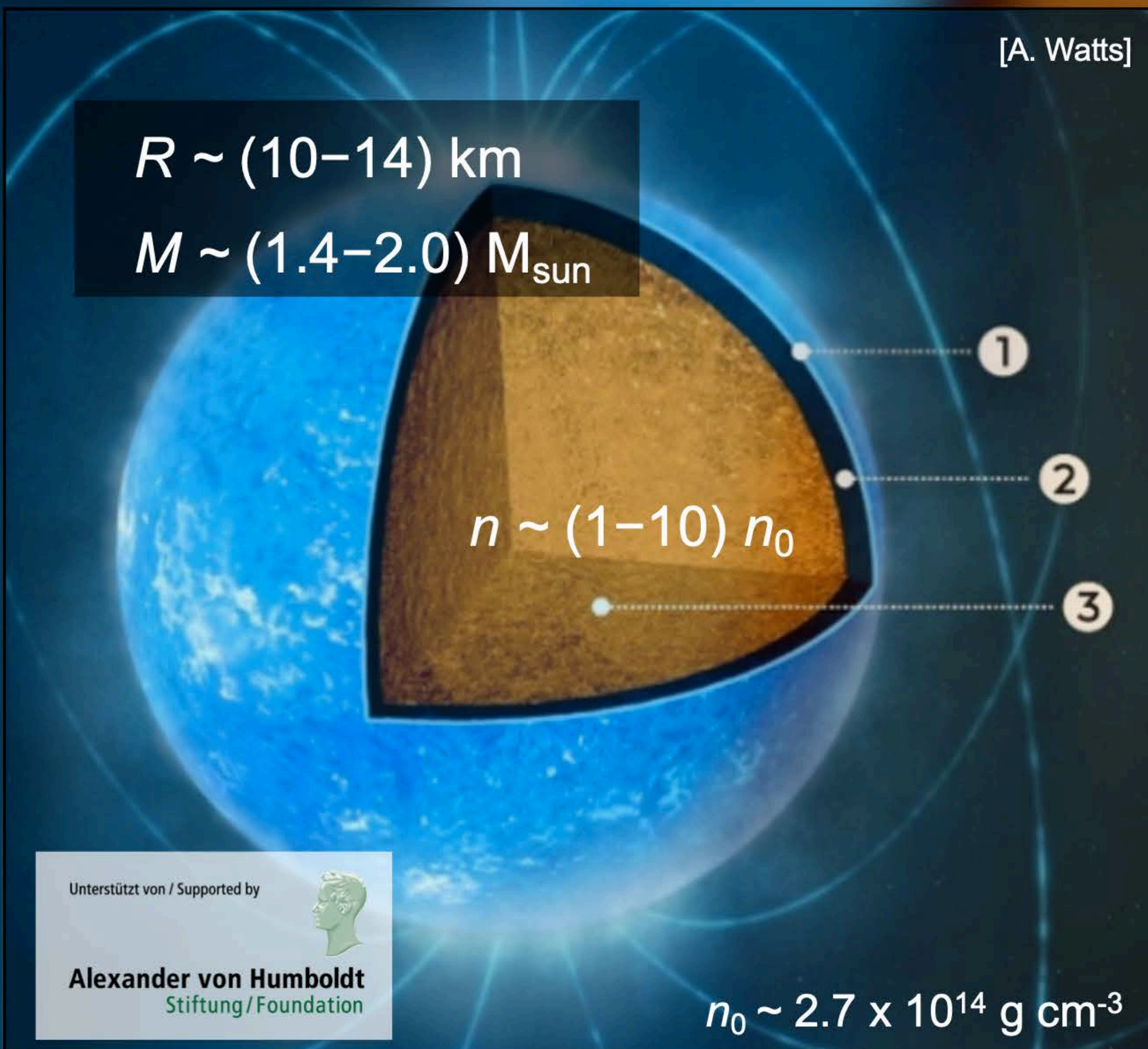


**Keywords:**

+ Chiral EFT
+ infinite nuclear matter
+ Bayesian UQ
+ symmetry energy
+ nuclear saturation
+ $N^3LO$ NN + 3N forces
+ …

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# Quantifying uncertainties in the nuclear-matter equation of state

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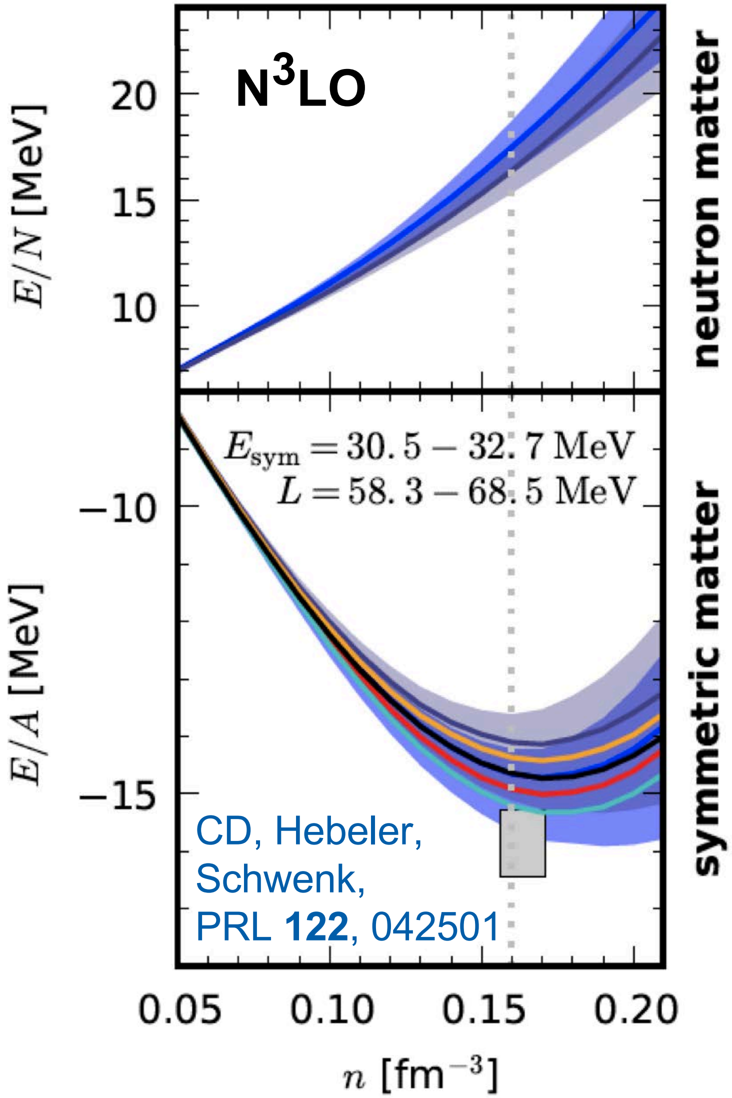
*e.g.*, Hebeler, Holt *et al.*, ARNP **65**, 457




**great progress** in predicting the **EOS** of infinite matter and the structure of **neutron stars** at densities $\lesssim n_0$

Hebeler, Lattimer *et al.*, APJ **773**, 11
Carbone, Rios *et al.*, PRC **88**, 044302
Hagen, Papenbrock *et al.* PRC **89**, 014319
Coraggio, Holt *et al.*, PRC **89**, 044321
Wellenhofer, Holt *et al.*, PRC **89**, 064009
CD, Carbone *et al.*, PRC **94,** 054307
Ekström, Hagen *et al.*, PRC **97**, 024332
Roggero, Mukherjee *et al.*, PRL **112**, 221103
CD, Hebeler *et al.*, PRL **122**, 042501
Lonardoni, Tews *et al.,* PRR **2**, 022033(R)
Piarulli, Bombaci *et al.*, PRC **101**, 045801
…

**But:** existing predictions **only** provided **rough estimates** for the with-density-growing **EFT truncation error**, and did *not* account for **correlations**

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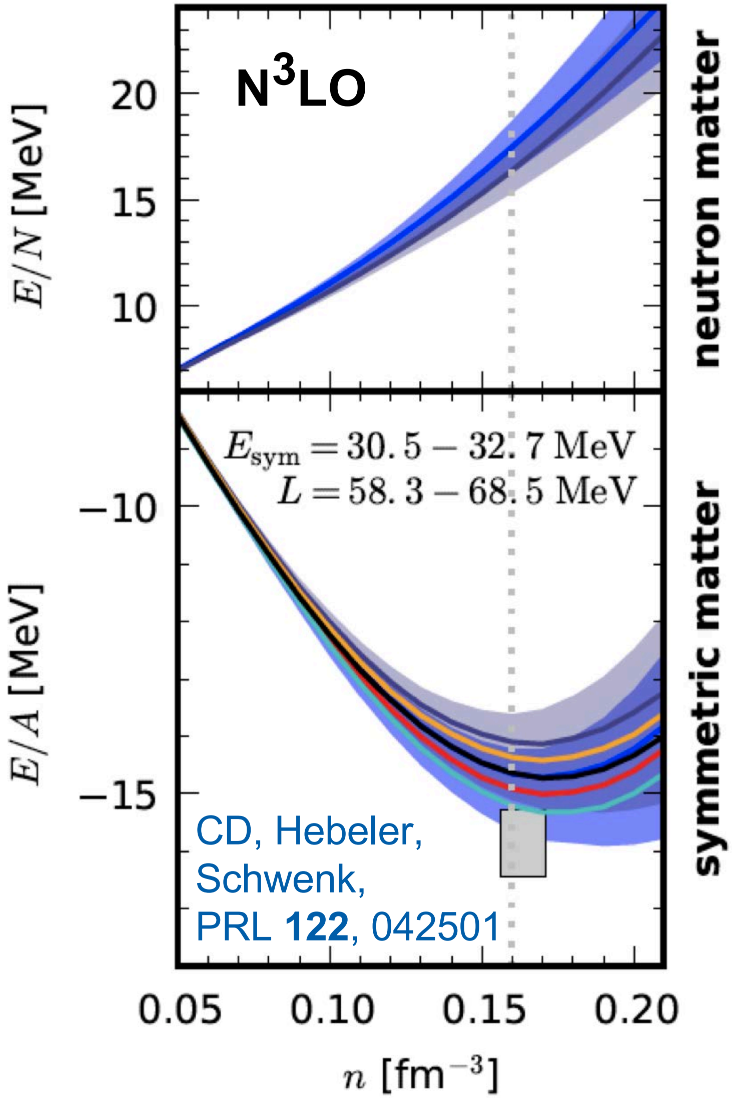


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**needed:** ***statistically* meaningful comparisons** between nuclear **theory** and recent **observational constraints**

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**But:** existing predictions **only** provided **rough estimates** for the with-density-growing **EFT truncation error**, and did *not* account for **correlations**

# Quantifying uncertainties in the nuclear-matter equation of state

## Direct detection of gravitational waves

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ligo.caltech.edu


multi-messenger astronomy

+ Virgo
+ GEO600
+ KAGRA
+ …

Binary Neutron-Star Merger

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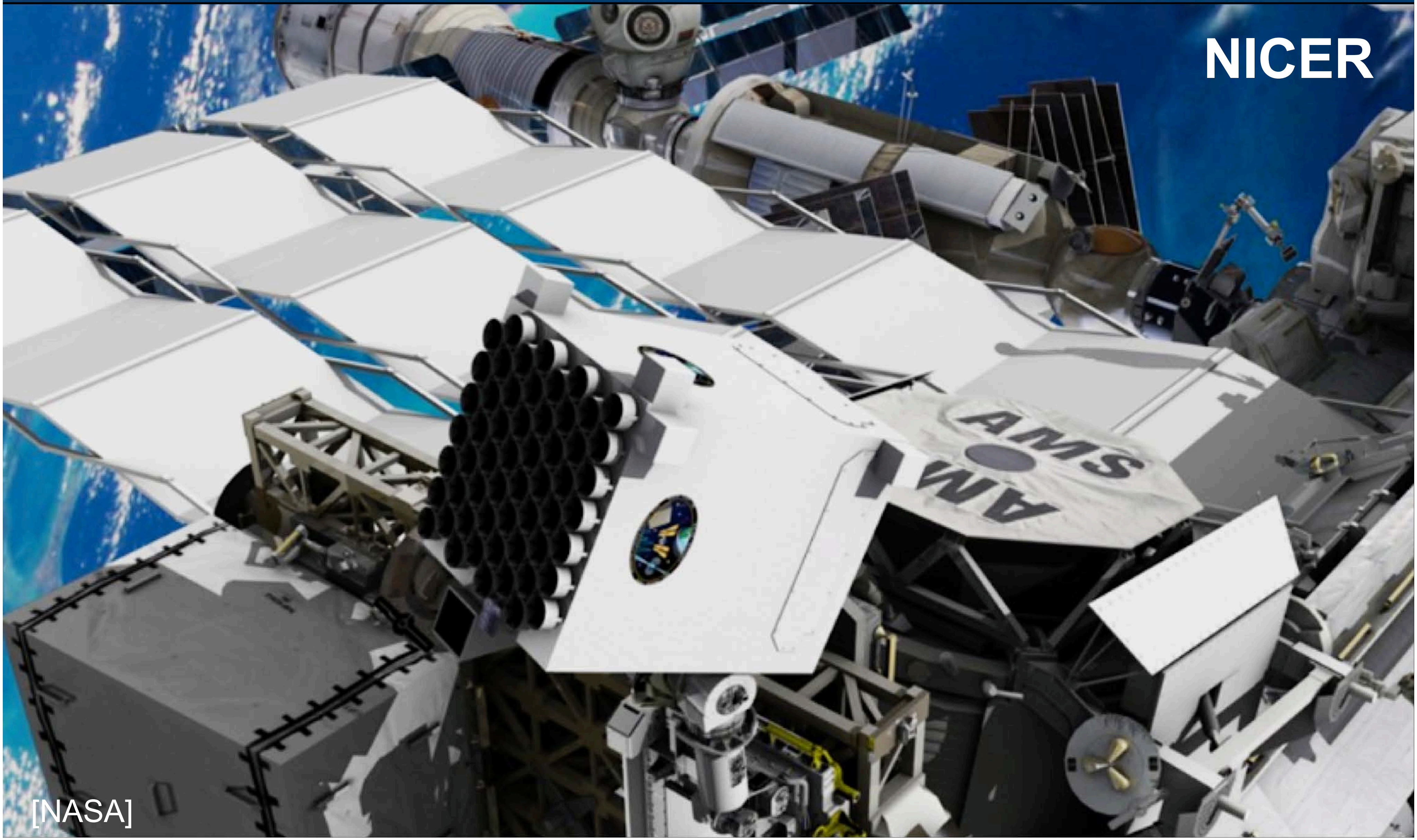
Observational constraints



# Quantifying uncertainties in the nuclear-matter equation of state

## Laboratory experiments

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frib.msu.edu

ISNET 2020 Information and Statistics in Nuclear Experiment and Theory (ISNET 8)

Facility for Rare Isotope Beams
at Michigan State University

**December 14 to 18, 2020 [virtual], and 2021**

**Local Organizing Committee:**

Pawel Danielewicz
Morten Hjorth-Jensen
Dean Lee
Witek Nazarewicz
Scott Pratt
Betty Tsang
Frederi Viens

# Quantifying uncertainties in the nuclear-matter equation of state

New framework for UQ of the infinite-matter EOS

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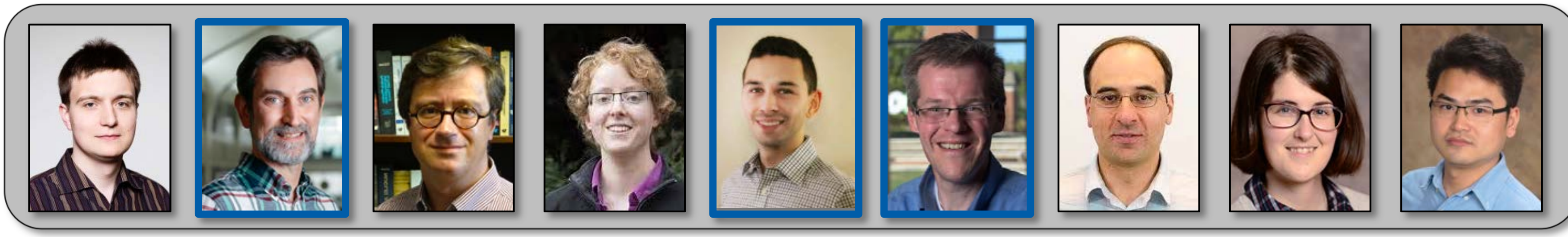
buqeye.github.io

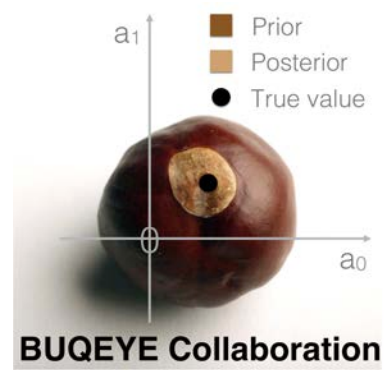


**CD, Furnstahl, Melendez, and Phillips**

*How well do we know the neutron-matter equation of state at the densities inside neutron stars? A Bayesian approach with correlated uncertainties*, [arXiv:2004.07232](https://arxiv.org/abs/2004.07232).

**CD, Melendez, Furnstahl, and Phillips**

*Effective Field Theory Convergence Pattern of Infinite Nuclear Matter*, [arXiv:2004.07805](https://arxiv.org/abs/2004.07805).




**B**ayesian
**U**ncertainty
**Q**uantification:
**E**rrors for
**Y**our
**E**FT

**UQ framework available at [https://buqeye.github.io](https://buqeye.github.io)**

# Quantifying uncertainties in the nuclear-matter equation of state

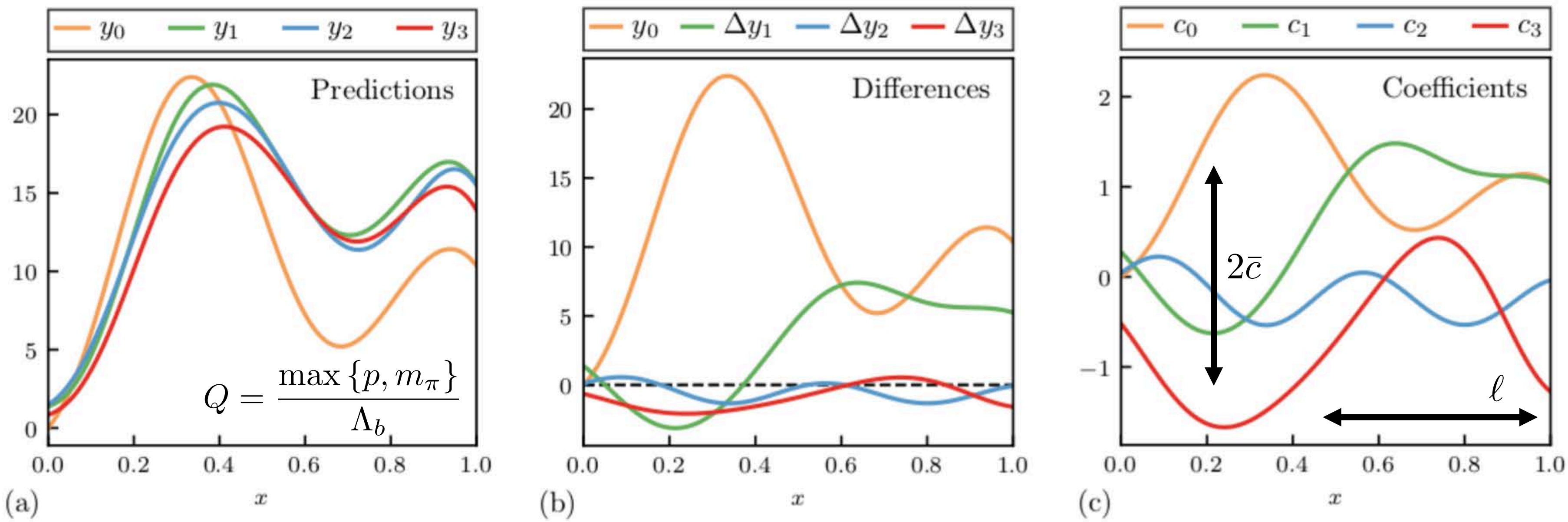
In a nutshell: EFT truncation-error model

Melendez, Furnstahl *et al.*, PRC **100**, 044001

**predict observable $y_k$ order by order in EFT**

$$\Delta y_n = y_n - y_{n-1}$$

**treat all $c_n$ as independent draws from a Gaussian Process**




$$y_k = y_{\text{ref}} \sum_{n=0}^{k} c_n Q^n$$

**infer EFT truncation error**

$$\delta y_k = y_{\text{ref}} \sum_{n=k+1}^{\infty} c_n Q^n$$

Note: $c_n$ are not the EFT's LEC

geometric sum

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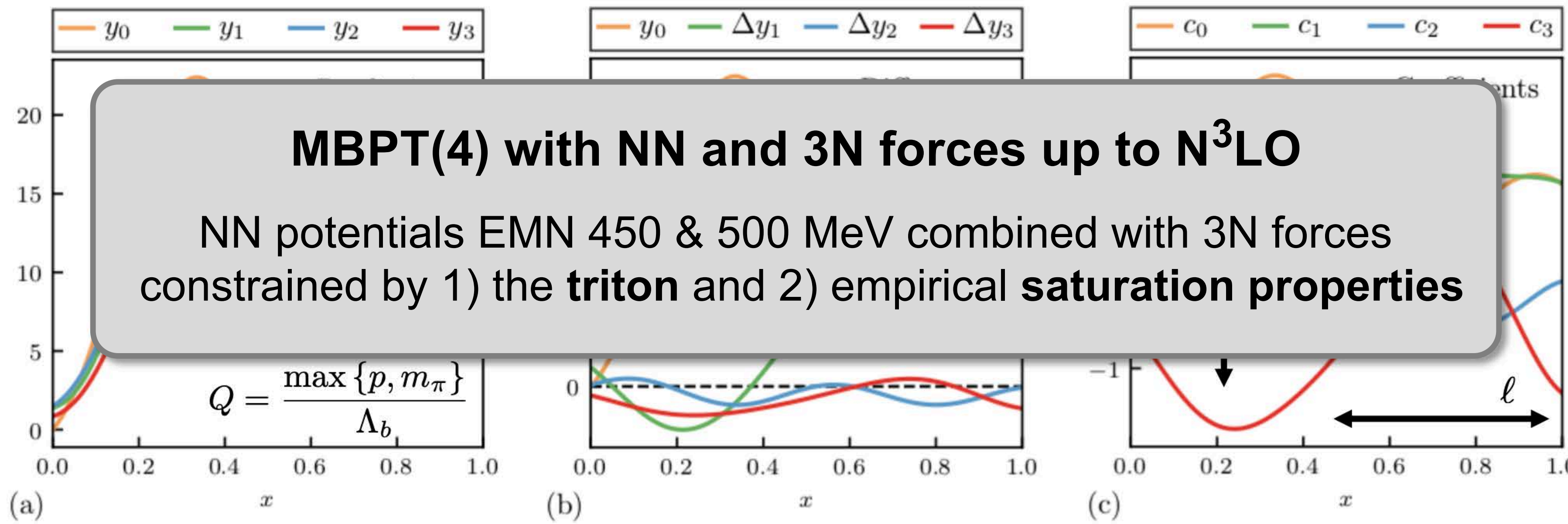


## MBPT(4) with NN and 3N forces up to N$^3$LO

NN potentials EMN 450 & 500 MeV combined with 3N forces constrained by 1) the **triton** and 2) empirical **saturation properties**

$$y_k = y_{\text{ref}} \sum_{n=0}^{k} c_n Q^n$$

**infer EFT truncation error**

$$\delta y_k = y_{\text{ref}} \sum_{n=k+1}^{\infty} c_n Q^n$$

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# Quantifying uncertainties in the nuclear-matter equation of state

A few physics questions

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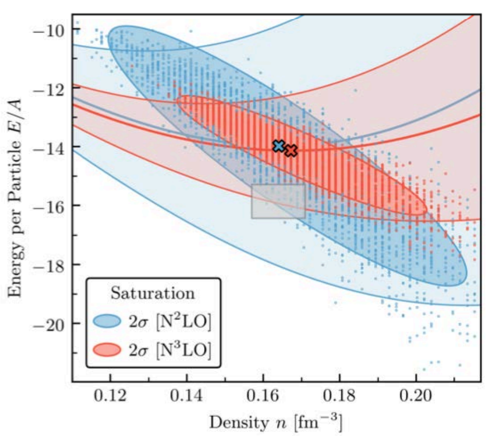
CD, Melendez *et al.*, arXiv:2004.07805

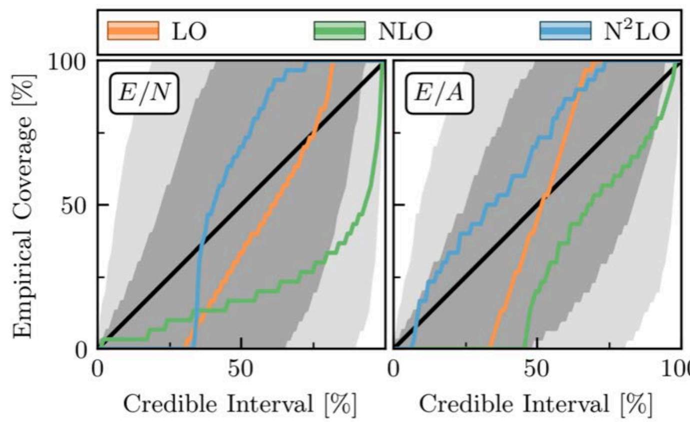
**Does** chiral EFT perform as advertised? If so, at which scale does it break down? If not, how to identify a more efficient EFT?

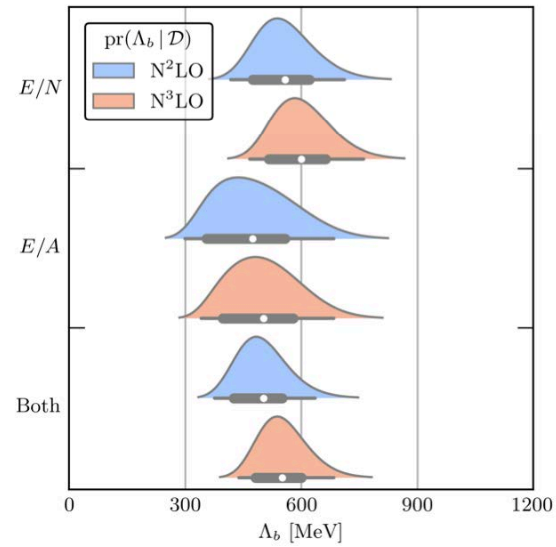
**How** can correlations be used to pinpoint observables?

**How well** can we reproduce the *empirical* saturation properties at the 1$\sigma$ level? Can we trust the uncertainty estimates?

**How** predictive is chiral EFT at 2–3 $n_0$?
And, what are the astrophysical implications?



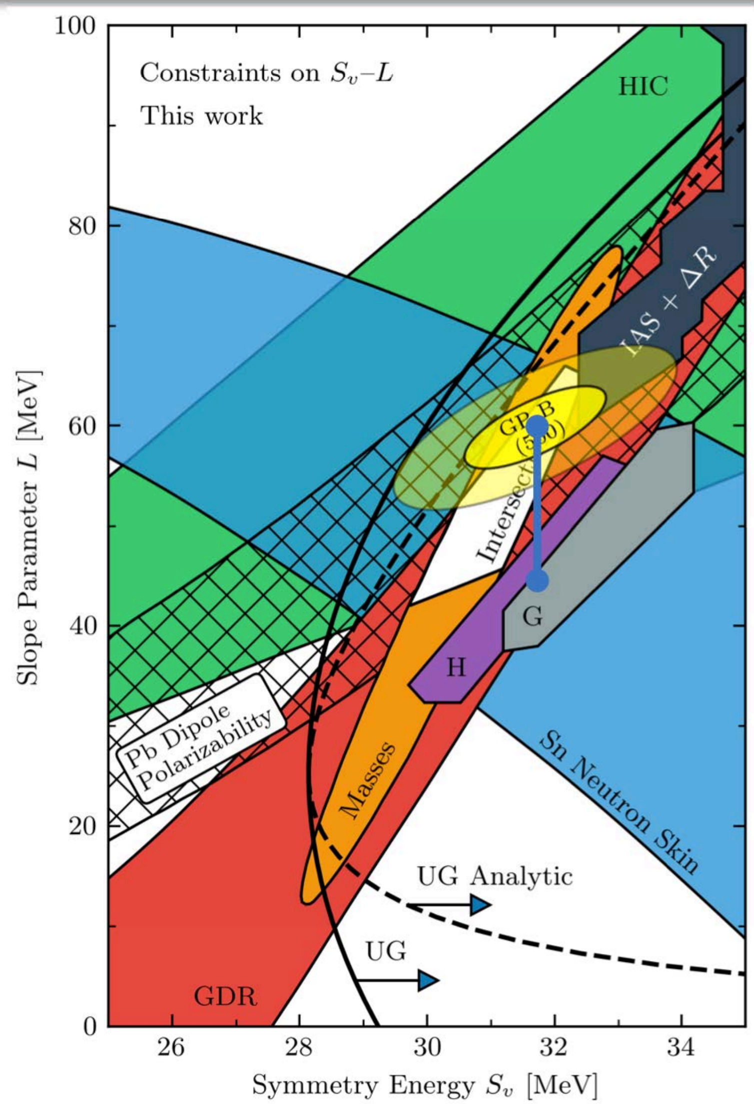



# Quantifying uncertainties in the nuclear-matter equation of state

## $S_v$–$L$ correlation (as compiled by Lattimer *et al.*)

CD, Furnstahl *et al.*, arXiv:2004.07805




$$S_2(n) \equiv S_v + \frac{L}{3}\left(\frac{n-n_0}{n_0}\right) + \ldots$$

**! Excellent agreement with experiment**

Lattimer and Lim, APJ **771**, 51

$$\mathrm{pr}(S_v, L \,|\, \mathcal{D}) = \int \mathrm{d}n_0\, \mathrm{pr}(S_2, L \,|\, n_0, \mathcal{D})\, \mathrm{pr}(n_0 \,|\, \mathcal{D})$$

$$\mathrm{pr}(n_0 \,|\, \mathcal{D}) \approx 0.17 \pm 0.01\,\mathrm{fm}^{-3}$$

$2\sigma$ ellipse (light yellow) is completely within the *conjectured* unitary gas limit

predicted range in $S_v$ **agrees** with other **theoretical constraints**; but ~15 MeV stronger density-dependence of $S_2(n_0)$

**two-dimensional Gaussian**

$$\begin{bmatrix}\mu_{S_v} \\ \mu_L\end{bmatrix} = \begin{bmatrix}31.7 \\ 59.8\end{bmatrix} \qquad \Sigma = \begin{bmatrix}1.24 & 3.27 \\ 3.27 & 16.95\end{bmatrix}$$

# Quantifying uncertainties in the nuclear-matter equation of state

## Pressure and speed of sound in PNM

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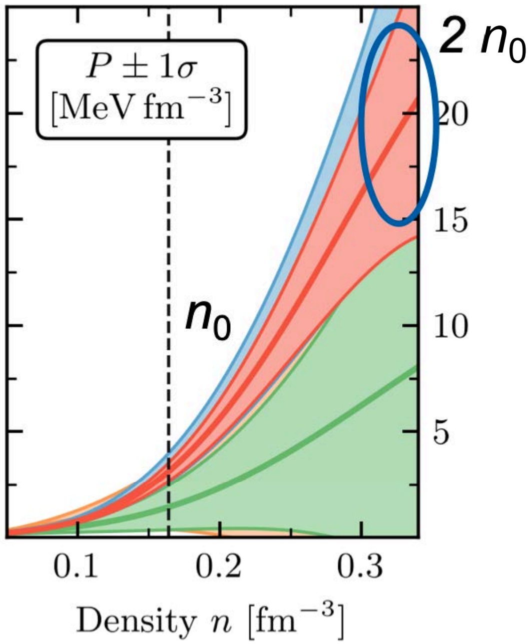
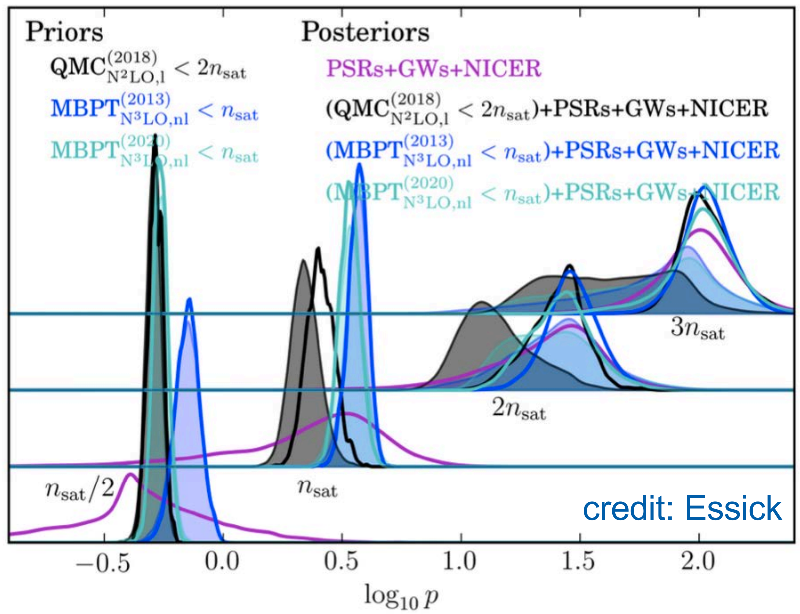
CD, Furnstahl *et al.*, arXiv:2004.07805

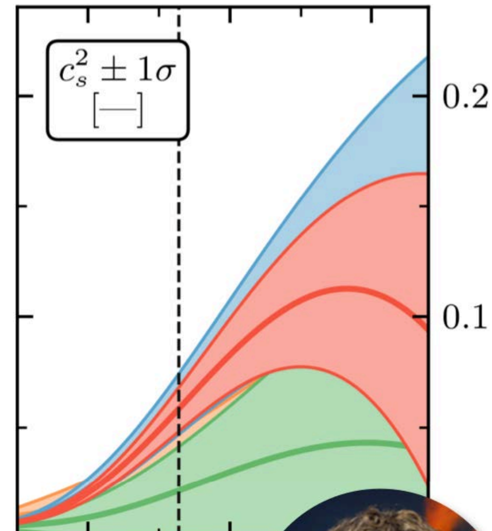
$$P(n = 0.32\,\mathrm{fm}^{-3}) \approx \begin{cases} 20 \pm 6\,\mathrm{MeV\,fm}^{-3} & \text{MBPT: nonlocal} \\ 15 \pm 5\,\mathrm{MeV\,fm}^{-3} & \text{QMC: local } V_{E,\mathbb{1}} \end{cases}$$

Tews *et al.* (2018)

$$c_s^2(n) = \frac{\partial P(n)}{\partial \varepsilon(n)}$$




**Direct astrophysical tests of chiral EFT at supranuclear densities**

Essick, Tews, Landry, Reddy, Holz, arXiv:2004.07744

$$P(n) = n^2 \frac{\mathrm{d}}{\mathrm{d}n} \frac{E}{N}(n)$$

LO NLO N$^2$LO N$^3$LO

# Quantifying uncertainties in the nuclear-matter equation of state

## Summary and outlook

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### 1 set a new standard for UQ in infinite-matter calculations

- correlations within *and* between observables are crucial for reliable UQ
- need for *statistically* meaningful comparisons between theory and observation
- efficiently quantify and propagate EOS uncertainties to derived quantities

### 2 *statistically robust* analysis of the EOS up to N$^3$LO

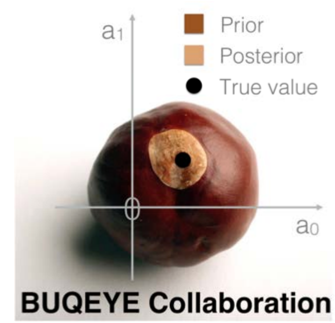
- excellent agreement of predicted $S_v$–$L$ correlation with experiment
- PNM and SNM show a regular EFT convergence pattern with increasing order
- extracted $\Lambda_b$ is consistent with NN scattering ● N$^2$LO coefficient may be an outlier

### 3 improved NN+3N potentials up to N$^3$LO are needed

- Hüther *et al.*, arXiv:1911.04955; Hoppe *et al.*, PRC **100**, 024318; …

### 4 full Bayesian UQ: MCMC for LECs & hyperparameters

- consistently include uncertainties in the LECs of chiral interactions
- compute nuclear saturation properties using Bayesian optimization





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R. Furnstahl **J. Melendez** **K. McElvain** D. Phillips