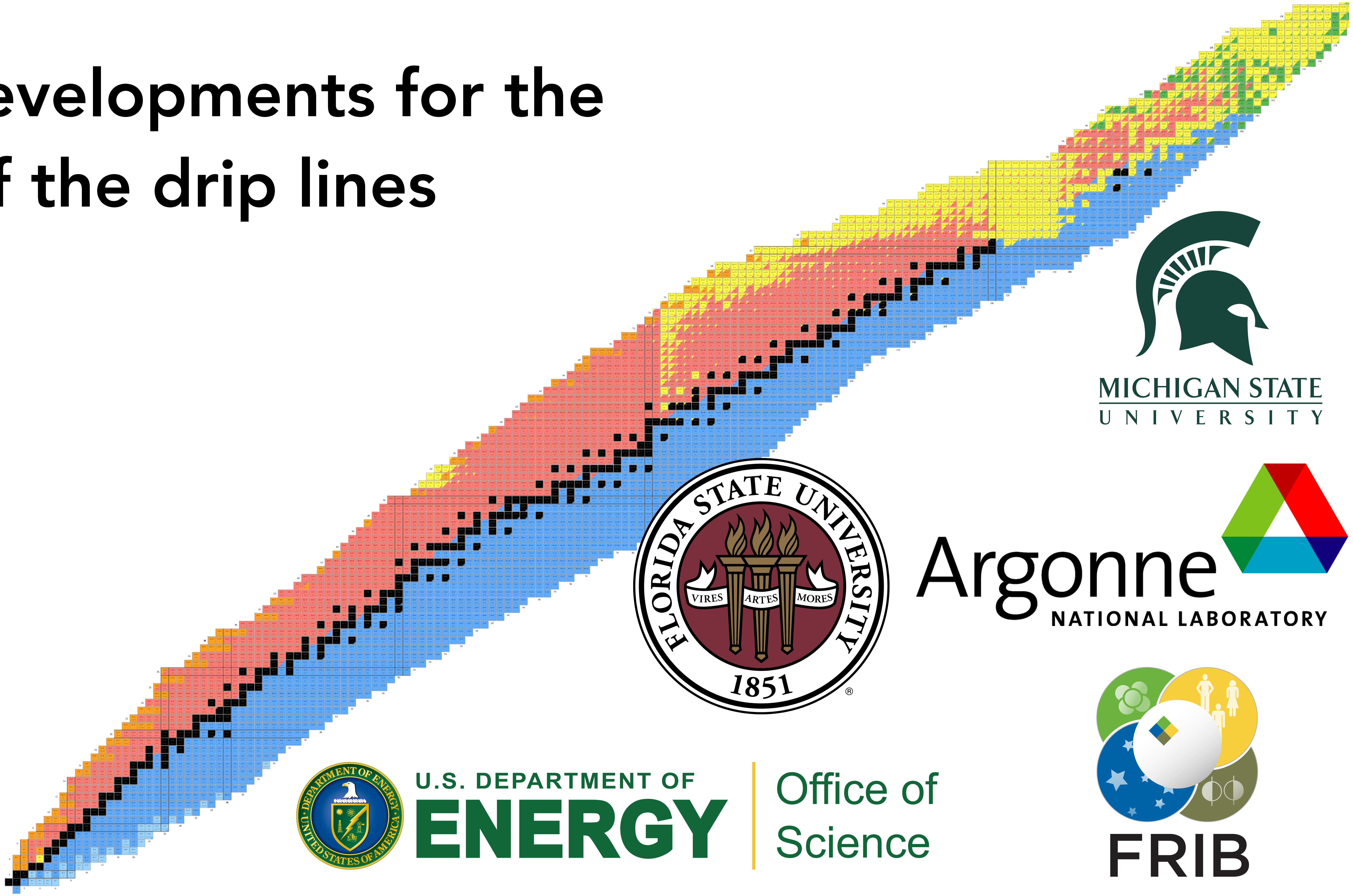
Theoretical developments for the exploration of the drip lines



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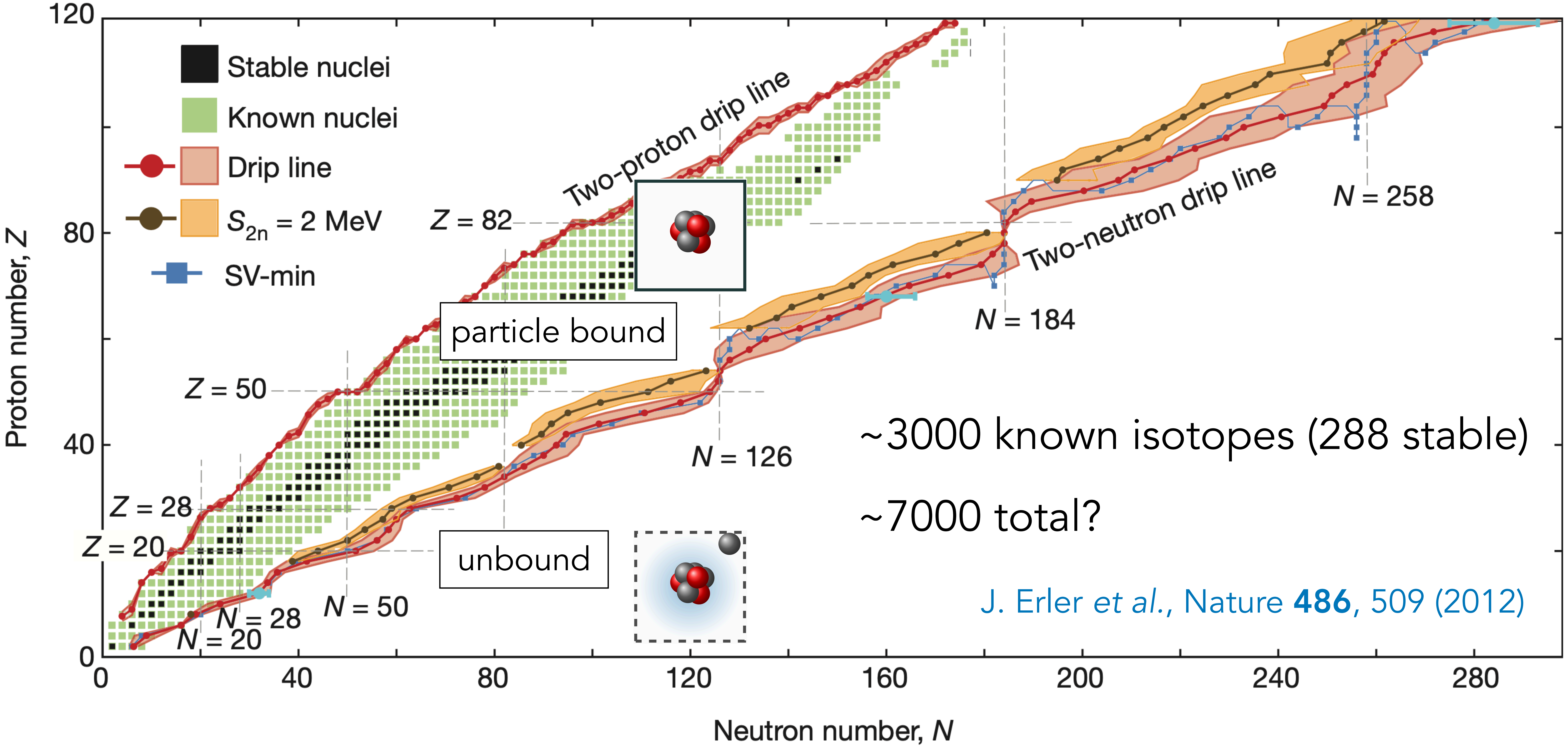
LECM, ANL
(online)

Aug. 9, 2022

DOE: DE-SC0013617 (Office of Nuclear Physics, FRIB Theory Alliance)

Motivation: test models & guide exp.

Limits of nuclear stability: How many isotopes can exist?



~3000 known isotopes (288 stable)

~7000 total?

J. Erler *et al.*, Nature **486**, 509 (2012)

Physics of exotic nuclei.

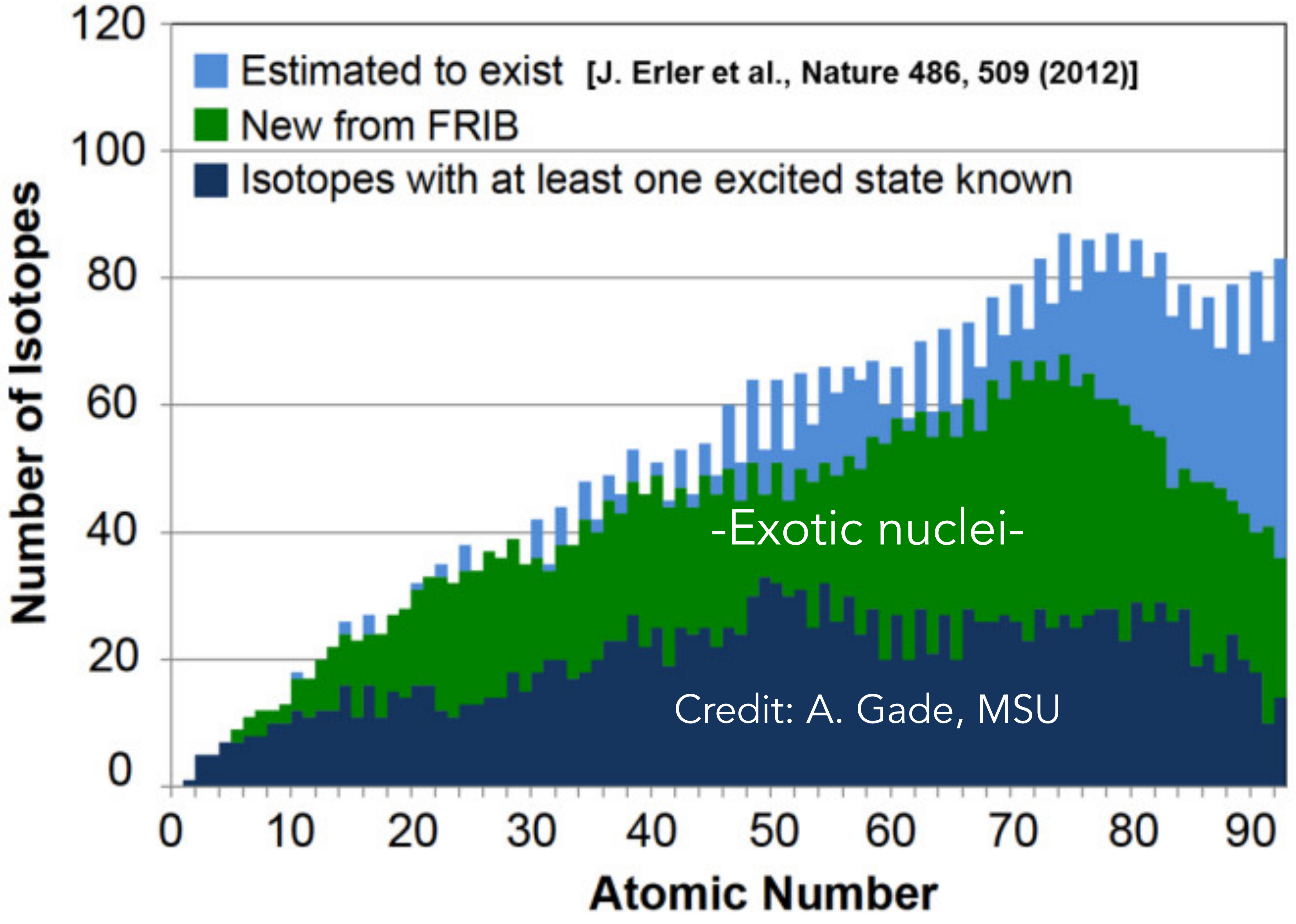
C. Johnson and K. Launey (eds.), J. Phys. G **47** 123001 (2020)

1. **Strong test for models:** either a nucleus exists, or it does not.
2. Exotic nuclei have **extreme N/Z ratios.**
3. Models can be tested using the **sensitivity of emergent properties.**

Theory is currently behind exp.

The FRIB era

- Nuclear structure, astrophysics, fundamental interactions, applications.

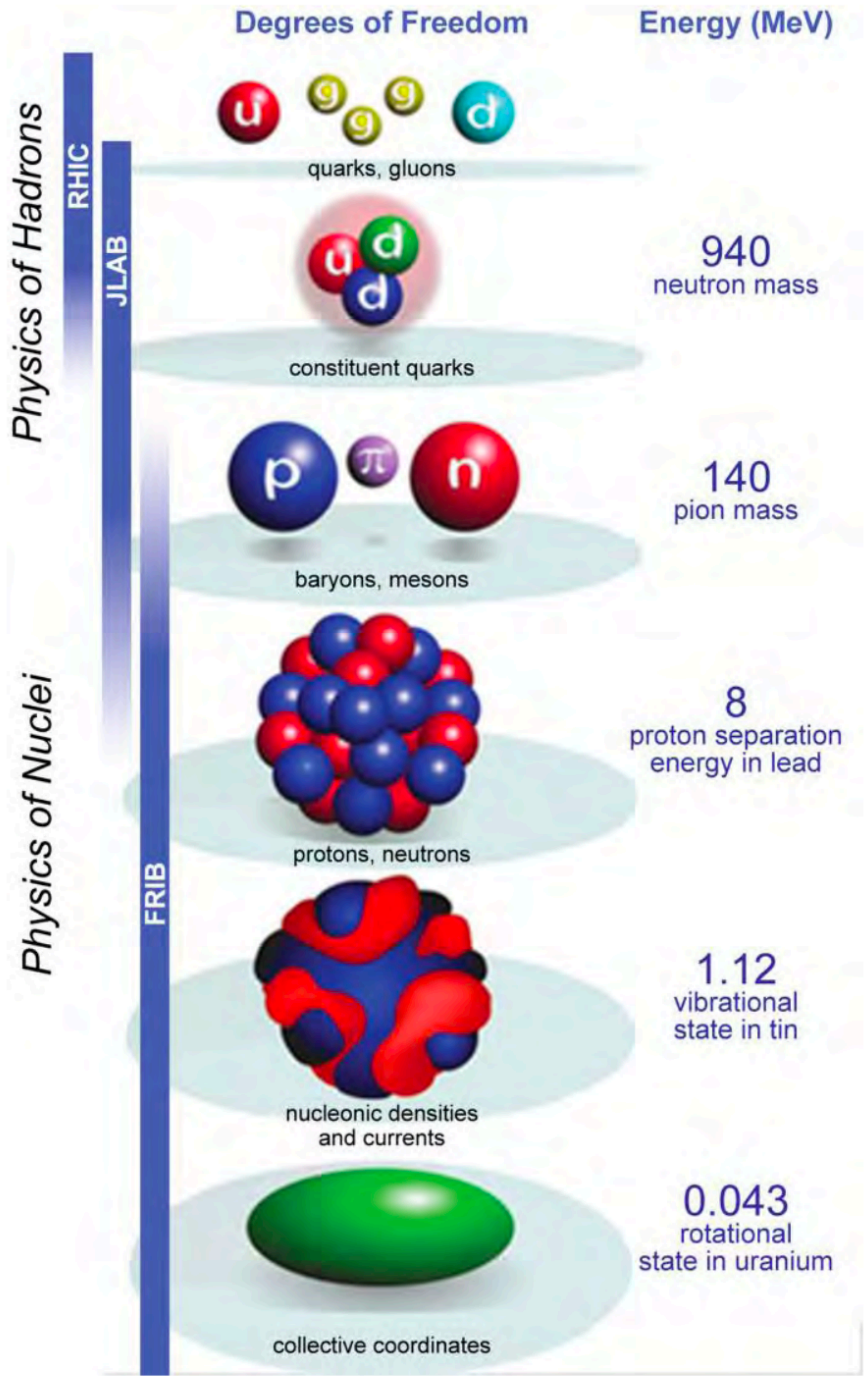


-Exotic nuclei-

Credit: A. Gade, MSU

- Theory needs to catch up!

Plenty of opportunities for the next +15 years!



FRIB - Long Range Plan

Gamow density matrix renormalization group

Configuration interaction + renormalization group.

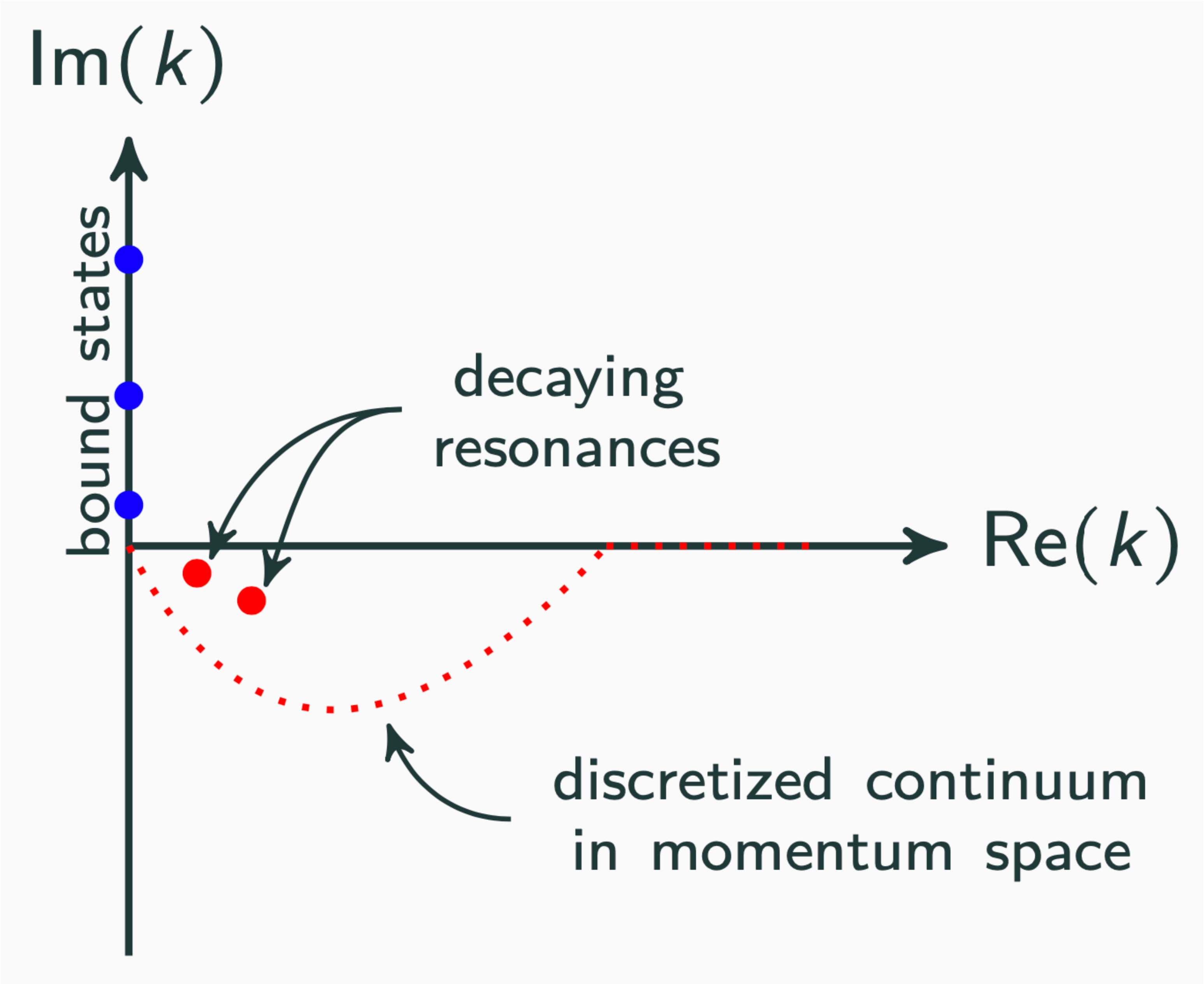
S. R. White, Phys. Rev. Lett. **69**, 2863 (1992)

$$|\Psi^{A,J^\pi}\rangle_1 = \sum_{a,b} C^{a}_{b,i=1} \{ |\mathrm{SD}^{f_\mathscr{A}}_{a}\rangle^{\mathscr{A}}_{0} \otimes |\mathrm{SD}^{f_\mathscr{B}}_{b}\rangle^{\mathscr{B}}_{1} \}^{A,J^\pi}$$

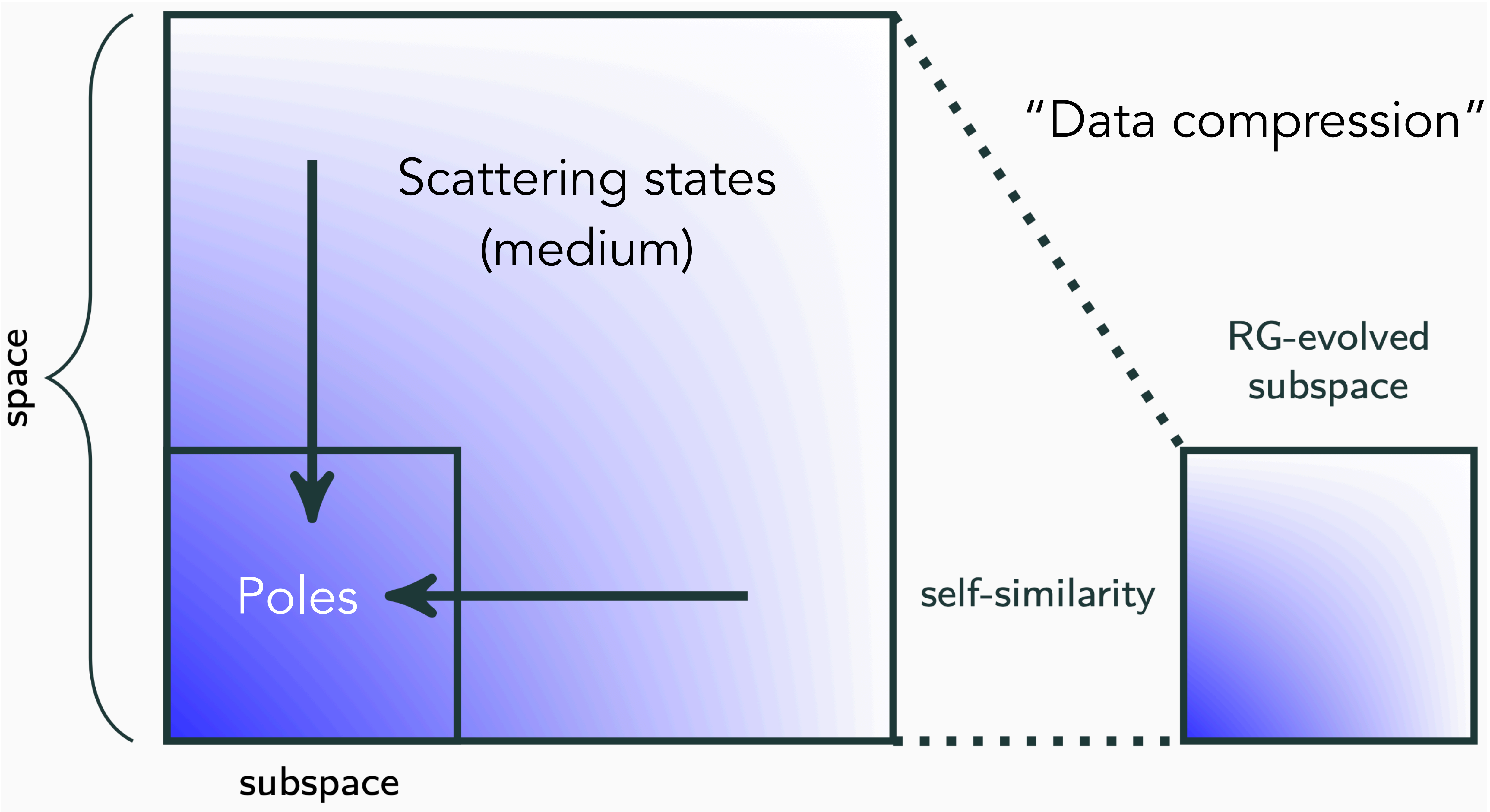
Reference space
= good approx.

Medium

$$\sum_{i=(b,r)} |k_i\rangle\langle\tilde{k}_i| + \int_{L^+} dk\, |k\rangle\langle\tilde{k}| = \hat{1}$$



T. Berggren, Nucl. Phys. A **109** 265 (1968)

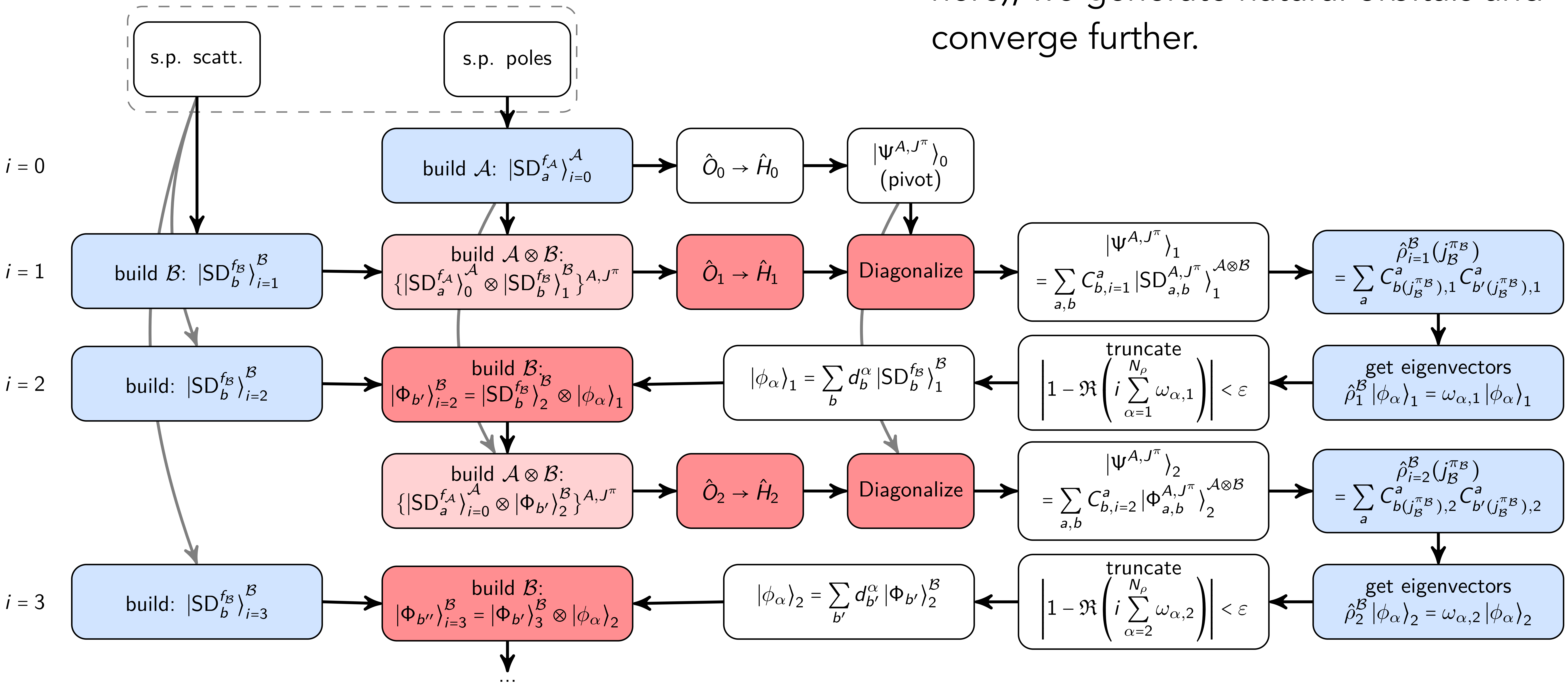


Gamow density matrix renormalization group

Only one DMRG code in the Berggren basis.

J. Rotureau *et al.*, Phys. Rev. Lett. **97**, 110603 (2006)

At the end of the "warm-up" phase (shown here), we generate natural orbitals and converge further.



s.p. scatt.

s.p. poles

$i = 0$

build $\mathcal{A}$: $|\mathrm{SD}_a^{f_\mathcal{A}}\rangle_{i=0}^{\mathcal{A}}$

$\hat{O}_0 \rightarrow \hat{H}_0$

$|\Psi^{A,J^\pi}\rangle_0$ (pivot)

$i = 1$

build $\mathcal{B}$: $|\mathrm{SD}_b^{f_\mathcal{B}}\rangle_{i=1}^{\mathcal{B}}$

build $\mathcal{A} \otimes \mathcal{B}$: $\{|\mathrm{SD}_a^{f_\mathcal{A}}\rangle_0^{\mathcal{A}} \otimes |\mathrm{SD}_b^{f_\mathcal{B}}\rangle_1^{\mathcal{B}}\}^{A,J^\pi}$

$\hat{O}_1 \rightarrow \hat{H}_1$

Diagonalize

$|\Psi^{A,J^\pi}\rangle_1 = \sum_{a,b} C_{b,i=1}^a |\mathrm{SD}_{a,b}^{A,J^\pi}\rangle_1^{\mathcal{A}\otimes\mathcal{B}}$

$\hat{\rho}_{i=1}^{\mathcal{B}}(j_\mathcal{B}^{\pi_\mathcal{B}}) = \sum_a C_{b(j_\mathcal{B}^{\pi_\mathcal{B}}),1}^a C_{b'(j_\mathcal{B}^{\pi_\mathcal{B}}),1}^a$

$i = 2$

build: $|\mathrm{SD}_b^{f_\mathcal{B}}\rangle_{i=2}^{\mathcal{B}}$

build $\mathcal{B}$: $|\Phi_{b'}\rangle_{i=2}^{\mathcal{B}} = |\mathrm{SD}_b^{f_\mathcal{B}}\rangle_2^{\mathcal{B}} \otimes |\phi_\alpha\rangle_1$

$|\phi_\alpha\rangle_1 = \sum_b d_b^\alpha |\mathrm{SD}_b^{f_\mathcal{B}}\rangle_1^{\mathcal{B}}$

truncate $\left|1 - \Re\left(i \sum_{\alpha=1}^{N_\rho} \omega_{\alpha,1}\right)\right| < \varepsilon$

get eigenvectors $\hat{\rho}_1^{\mathcal{B}} |\phi_\alpha\rangle_1 = \omega_{\alpha,1} |\phi_\alpha\rangle_1$

build $\mathcal{A} \otimes \mathcal{B}$: $\{|\mathrm{SD}_a^{f_\mathcal{A}}\rangle_{i=0}^{\mathcal{A}} \otimes |\Phi_{b'}\rangle_2^{\mathcal{B}}\}^{A,J^\pi}$

$\hat{O}_2 \rightarrow \hat{H}_2$

Diagonalize

$|\Psi^{A,J^\pi}\rangle_2 = \sum_{a,b} C_{b,i=2}^a |\Phi_{a,b}^{A,J^\pi}\rangle_2^{\mathcal{A}\otimes\mathcal{B}}$

$\hat{\rho}_{i=2}^{\mathcal{B}}(j_\mathcal{B}^{\pi_\mathcal{B}}) = \sum_a C_{b(j_\mathcal{B}^{\pi_\mathcal{B}}),2}^a C_{b'(j_\mathcal{B}^{\pi_\mathcal{B}}),2}^a$

$i = 3$

build: $|\mathrm{SD}_b^{f_\mathcal{B}}\rangle_{i=3}^{\mathcal{B}}$

build $\mathcal{B}$: $|\Phi_{b''}\rangle_{i=3}^{\mathcal{B}} = |\Phi_{b'}\rangle_3^{\mathcal{B}} \otimes |\phi_\alpha\rangle_2$

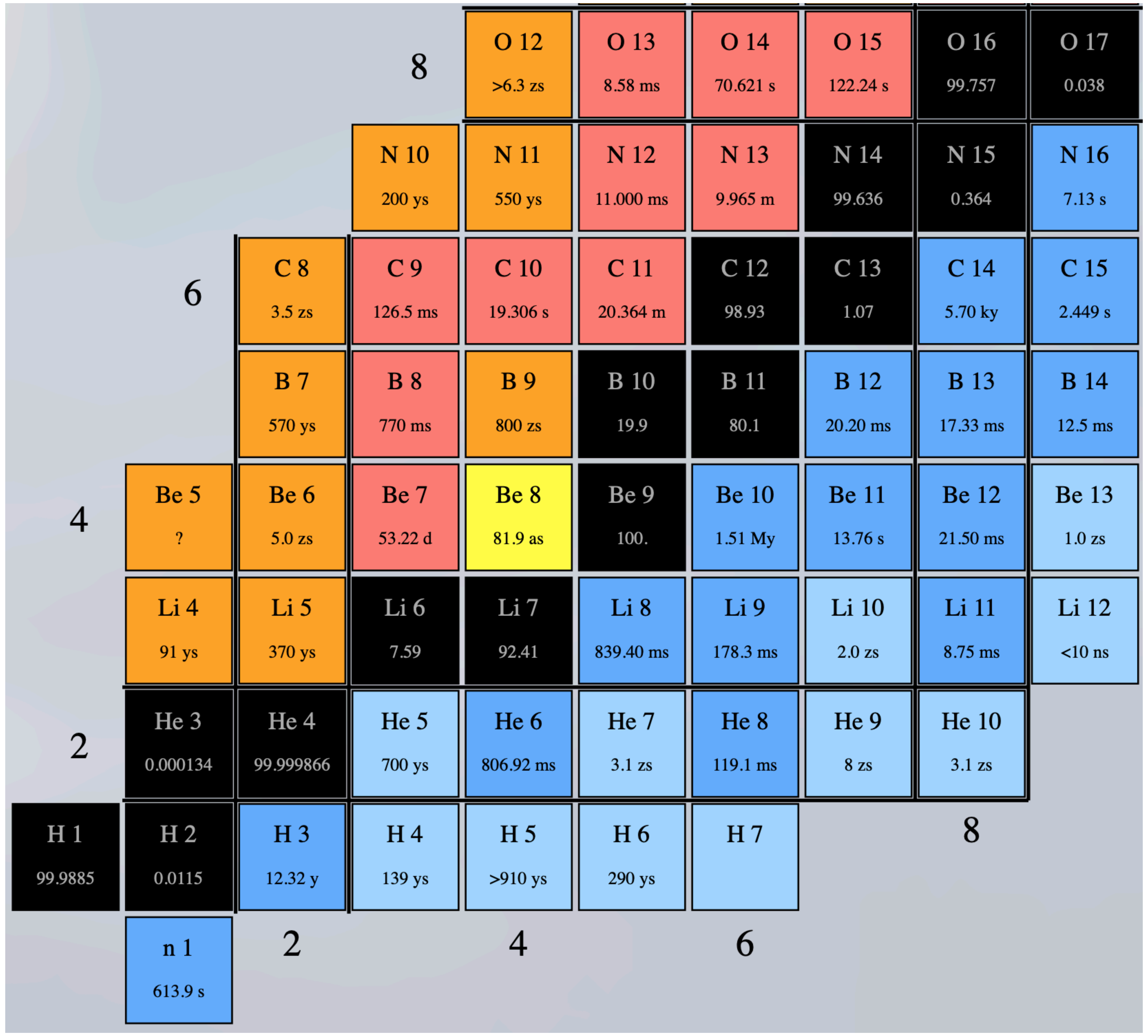
$|\phi_\alpha\rangle_2 = \sum_{b'} d_{b'}^\alpha |\Phi_{b'}\rangle_2^{\mathcal{B}}$

truncate $\left|1 - \Re\left(i \sum_{\alpha=2}^{N_\rho} \omega_{\alpha,2}\right)\right| < \varepsilon$

get eigenvectors $\hat{\rho}_2^{\mathcal{B}} |\phi_\alpha\rangle_2 = \omega_{\alpha,2} |\phi_\alpha\rangle_2$

…

Ab initio DMRG for light exotic nuclei



			8	O 12 >6.3 zs	O 13 8.58 ms	O 14 70.621 s	O 15 122.24 s	O 16 99.757	O 17 0.038
			N 10 200 ys	N 11 550 ys	N 12 11.000 ms	N 13 9.965 m	N 14 99.636	N 15 0.364	N 16 7.13 s
	6	C 8 3.5 zs	C 9 126.5 ms	C 10 19.306 s	C 11 20.364 m	C 12 98.93	C 13 1.07	C 14 5.70 ky	C 15 2.449 s
		B 7 570 ys	B 8 770 ms	B 9 800 zs	B 10 19.9	B 11 80.1	B 12 20.20 ms	B 13 17.33 ms	B 14 12.5 ms
4	Be 5 ?	Be 6 5.0 zs	Be 7 53.22 d	Be 8 81.9 as	Be 9 100.	Be 10 1.51 My	Be 11 13.76 s	Be 12 21.50 ms	Be 13 1.0 zs
	Li 4 91 ys	Li 5 370 ys	Li 6 7.59	Li 7 92.41	Li 8 839.40 ms	Li 9 178.3 ms	Li 10 2.0 zs	Li 11 8.75 ms	Li 12 <10 ns
2	He 3 0.000134	He 4 99.999866	He 5 700 ys	He 6 806.92 ms	He 7 3.1 zs	He 8 119.1 ms	He 9 8 zs	He 10 3.1 zs	
H 1 99.9885	H 2 0.0115	H 3 12.32 y	H 4 139 ys	H 5 >910 ys	H 6 290 ys	H 7		8	
	n 1 613.9 s	2		4		6			

Up to A=12, there are 24 bound nuclei (including 5 halos), and 21 (known) unbound nuclei.

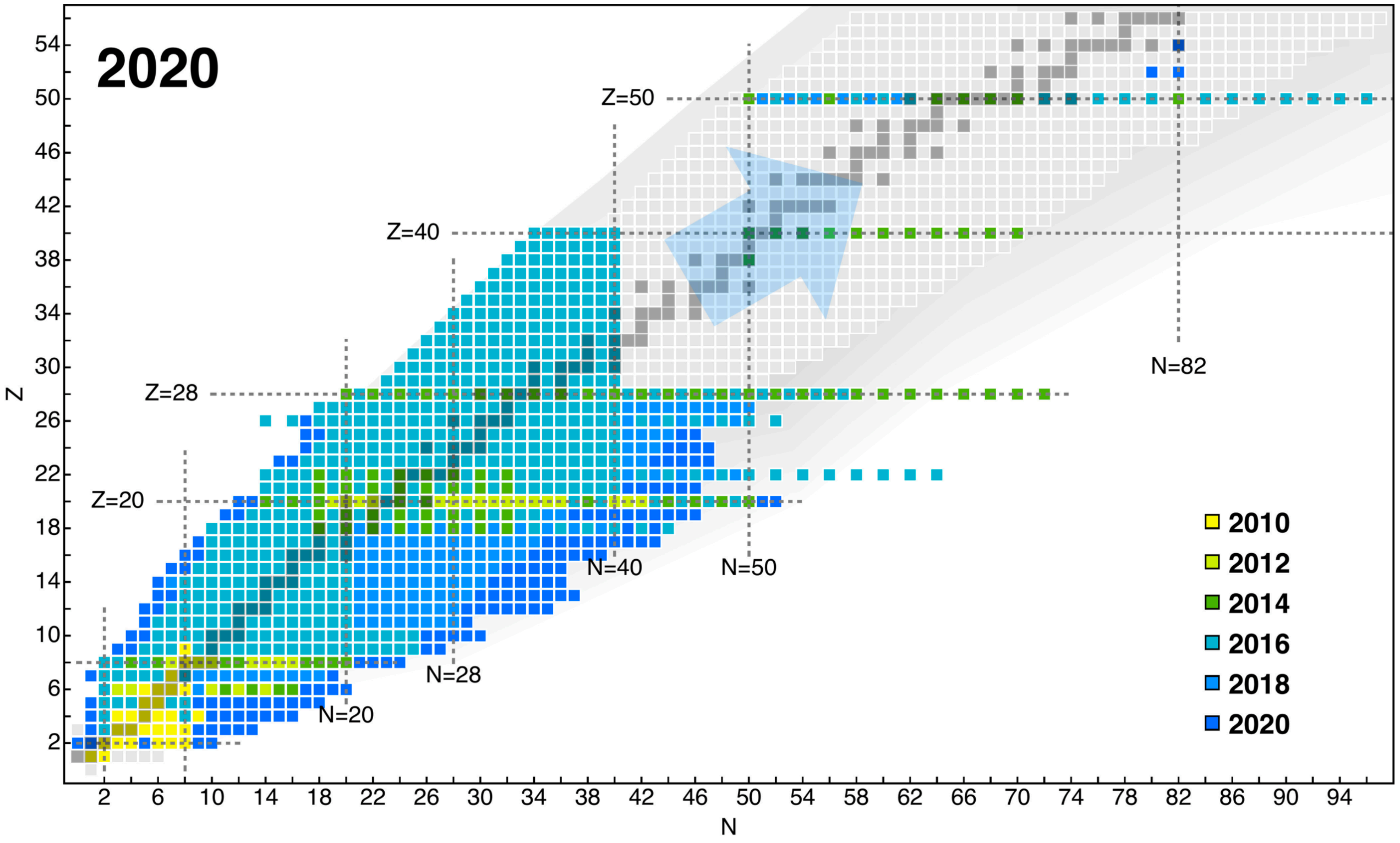
Goal: Perform systematic calculations to test nuclear forces on light exotic nuclei.

Both theoretical and computational developments needed:

- Multi-step DMRG method using natural orbitals.
- Optimize I/O and matrix elements calculations.

Ab initio IM-DMRG for medium-mass nuclei

H. Hergert, Front. Phys. **8**, 379 (2020)



Combine the (multi-reference) in-medium similarity renormalization group (IMSRG) method with the DMRG.

Goal: Extend the reach of ab initio calculations, access excited states, and capture static and dynamic correlations.

Pre-diagonalize the Hamiltonian using IMSRG and a DMRG reference state, then finish the renormalization using DMRG.

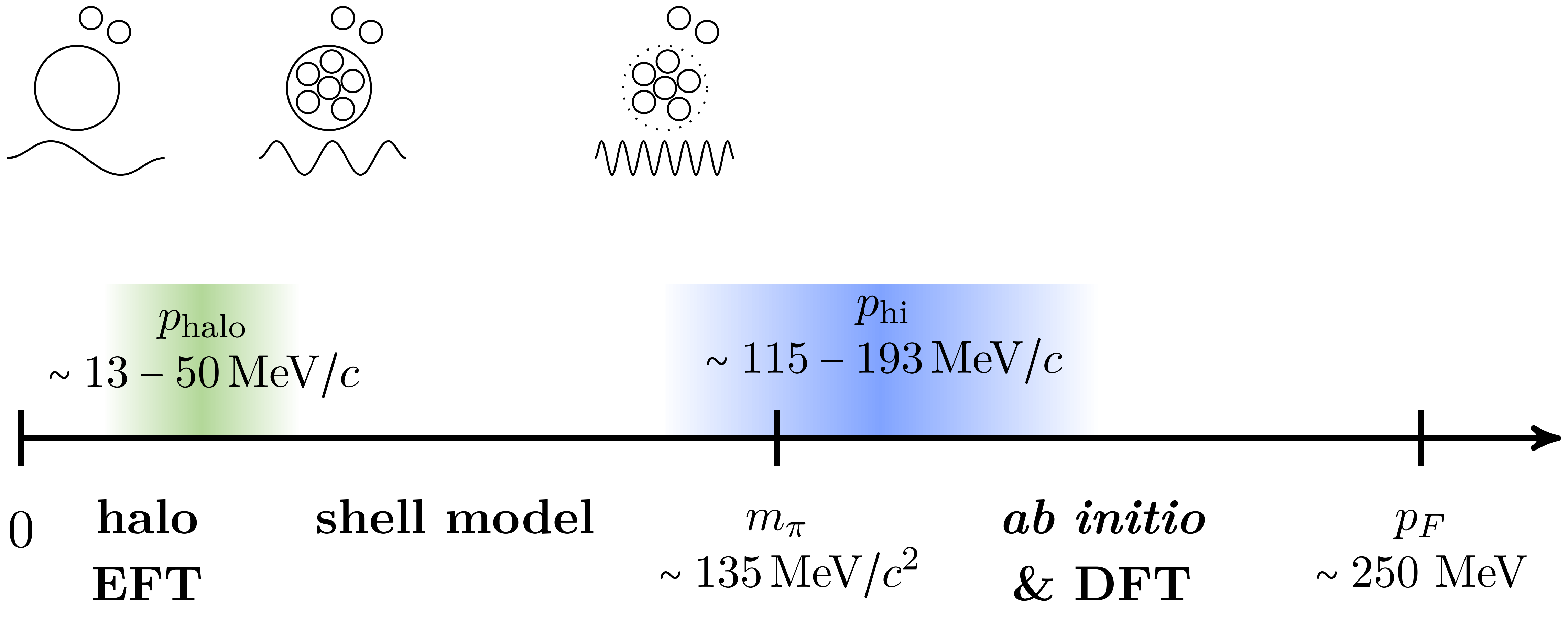
Effective theory of the nuclear shell model

3-body halo-EFT is too restrictive to explain the SM:

$$\frac{\Lambda_{\mathbf{lo}}}{\Lambda_{\mathbf{hi}}} = \frac{S_a(A)}{\min[E_x(A_c), S_{n,p}(A_c)]}$$

$\Lambda_{\mathrm{lo}}/\Lambda_{\mathrm{hi}} \approx 2.0$
(18O with 16O core)

Goal: Build an effective theory of the SM to connect with ab initio and guide experiment.



Effective separation of scales:

- Remove one valence nucleon:

$E_v \approx 0.2 - 8.5$ MeV (→ E/A when A large).

- Remove one core nucleon:

$E_c \approx 12 - 20$ MeV (typical SM cores).

In exotic nuclei $E_v/E_c \ll 1$.

$\lvert E_v/E_c \rvert \approx 0.78$	(18O with 16O core)
$\lvert E_v/E_c \rvert \approx 0.64$	(26O with 16O core)
$\lvert E_v/E_c \rvert \approx 0.001$	(26O with 24O core)

$p_c \approx m_\pi c$ → pionless-EFT, but… too complicated order-by-order*.

→ Re-expansion around a finite momentum.

Relevance for the FRIB-PAC1

21055:
intruder states,
deformation

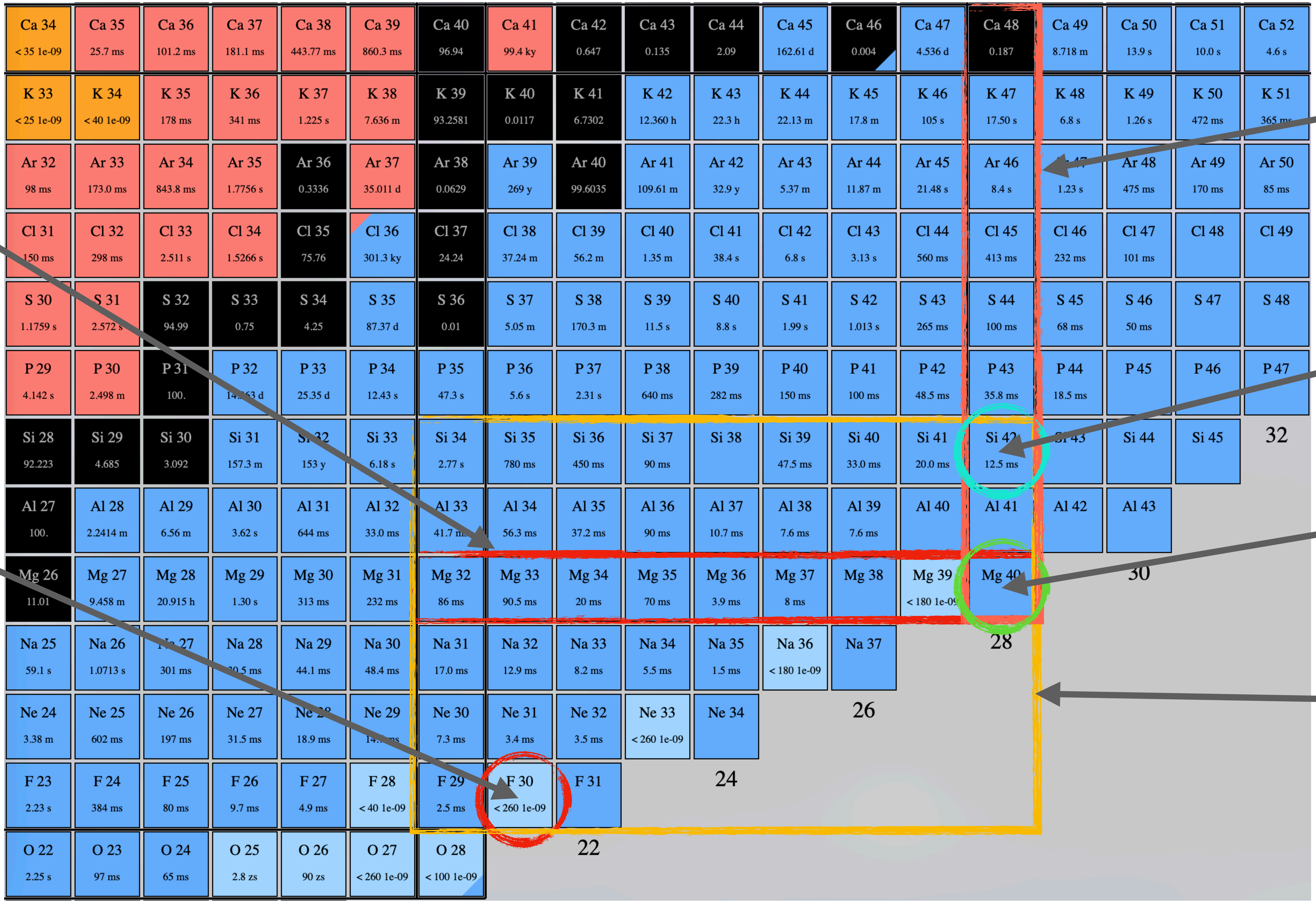
K. Fossez *et al.*,
PRC **94**, 054302 (2016)

21016: decay
spectrum

**(Contributed to
the proposal)**

X-Y. Luo, K. Fossez *et al.*,
PRC **104**, 014307 (2021)

K. Fossez *et al.*,
Submitted to PRC (2022)



Ca 34 < 35 1e-09	Ca 35 25.7 ms	Ca 36 101.2 ms	Ca 37 181.1 ms	Ca 38 443.77 ms	Ca 39 860.3 ms	Ca 40 96.94	Ca 41 99.4 ky	Ca 42 0.647	Ca 43 0.135	Ca 44 2.09	Ca 45 162.61 d	Ca 46 0.004	Ca 47 4.536 d	Ca 48 0.187	Ca 49 8.718 m	Ca 50 13.9 s	Ca 51 10.0 s	Ca 52 4.6 s
K 33 < 25 1e-09	K 34 < 40 1e-09	K 35 178 ms	K 36 341 ms	K 37 1.225 s	K 38 7.636 m	K 39 93.2581	K 40 0.0117	K 41 6.7302	K 42 12.360 h	K 43 22.3 h	K 44 22.13 m	K 45 17.8 m	K 46 105 s	K 47 17.50 s	K 48 6.8 s	K 49 1.26 s	K 50 472 ms	K 51 365 ms
Ar 32 98 ms	Ar 33 173.0 ms	Ar 34 843.8 ms	Ar 35 1.7756 s	Ar 36 0.3336	Ar 37 35.011 d	Ar 38 0.0629	Ar 39 269 y	Ar 40 99.6035	Ar 41 109.61 m	Ar 42 32.9 y	Ar 43 5.37 m	Ar 44 11.87 m	Ar 45 21.48 s	Ar 46 8.4 s	Ar 47 1.23 s	Ar 48 475 ms	Ar 49 170 ms	Ar 50 85 ms
Cl 31 150 ms	Cl 32 298 ms	Cl 33 2.511 s	Cl 34 1.5266 s	Cl 35 75.76	Cl 36 301.3 ky	Cl 37 24.24	Cl 38 37.24 m	Cl 39 56.2 m	Cl 40 1.35 m	Cl 41 38.4 s	Cl 42 6.8 s	Cl 43 3.13 s	Cl 44 560 ms	Cl 45 413 ms	Cl 46 232 ms	Cl 47 101 ms	Cl 48	Cl 49
S 30 1.1759 s	S 31 2.572 s	S 32 94.99	S 33 0.75	S 34 4.25	S 35 87.37 d	S 36 0.01	S 37 5.05 m	S 38 170.3 m	S 39 11.5 s	S 40 8.8 s	S 41 1.99 s	S 42 1.013 s	S 43 265 ms	S 44 100 ms	S 45 68 ms	S 46 50 ms	S 47	S 48
P 29 4.142 s	P 30 2.498 m	P 31 100.	P 32 14.263 d	P 33 25.35 d	P 34 12.43 s	P 35 47.3 s	P 36 5.6 s	P 37 2.31 s	P 38 640 ms	P 39 282 ms	P 40 150 ms	P 41 100 ms	P 42 48.5 ms	P 43 35.8 ms	P 44 18.5 ms	P 45	P 46	P 47
Si 28 92.223	Si 29 4.685	Si 30 3.092	Si 31 157.3 m	Si 32 153 y	Si 33 6.18 s	Si 34 2.77 s	Si 35 780 ms	Si 36 450 ms	Si 37 90 ms	Si 38	Si 39 47.5 ms	Si 40 33.0 ms	Si 41 20.0 ms	Si 42 12.5 ms	Si 43	Si 44	Si 45	32
Al 27 100.	Al 28 2.2414 m	Al 29 6.56 m	Al 30 3.62 s	Al 31 644 ms	Al 32 33.0 ms	Al 33 41.7 ms	Al 34 56.3 ms	Al 35 37.2 ms	Al 36 90 ms	Al 37 10.7 ms	Al 38 7.6 ms	Al 39 7.6 ms	Al 40	Al 41	Al 42	Al 43		
Mg 26 11.01	Mg 27 9.458 m	Mg 28 20.915 h	Mg 29 1.30 s	Mg 30 313 ms	Mg 31 232 ms	Mg 32 86 ms	Mg 33 90.5 ms	Mg 34 20 ms	Mg 35 70 ms	Mg 36 3.9 ms	Mg 37 8 ms	Mg 38	Mg 39 < 180 1e-09	Mg 40		30		
Na 25 59.1 s	Na 26 1.0713 s	Na 27 301 ms	Na 28 30.5 ms	Na 29 44.1 ms	Na 30 48.4 ms	Na 31 17.0 ms	Na 32 12.9 ms	Na 33 8.2 ms	Na 34 5.5 ms	Na 35 1.5 ms	Na 36 < 180 1e-09	Na 37		28				
Ne 24 3.38 m	Ne 25 602 ms	Ne 26 197 ms	Ne 27 31.5 ms	Ne 28 18.9 ms	Ne 29 14.[illegible] ms	Ne 30 7.3 ms	Ne 31 3.4 ms	Ne 32 3.5 ms	Ne 33 < 260 1e-09	Ne 34		26						
F 23 2.23 s	F 24 384 ms	F 25 80 ms	F 26 9.7 ms	F 27 4.9 ms	F 28 < 40 1e-09	F 29 2.5 ms	F 30 < 260 1e-09	F 31		24								
O 22 2.25 s	O 23 97 ms	O 24 65 ms	O 25 2.8 zs	O 26 90 zs	O 27 < 260 1e-09	O 28 < 100 1e-09		22										

21004: shell
evolution

21009:
structure,
shapes, weak
binding

21001:
deformation

21004: halo
structure

21062: beta
decay
spectroscopy

Summary

In the FRIB era, the tension between experiment and theory at the drip lines needs to be addressed.

Nuclear forces need to be tested in light exotic nuclei to improve their reliability in the medium-mass region → **ab initio DMRG.**

Ab initio theory needs to be extended to larger nuclei, to give access to excited states, and to capture static and dynamic correlations → **in-medium DMRG.**

Coming experiments at FRIB need theoretical support and precise predictions at the drip lines → **effective theory of the shell model.**

Thank you for your attention!

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