Few-Body Perspectives in Finite Volume

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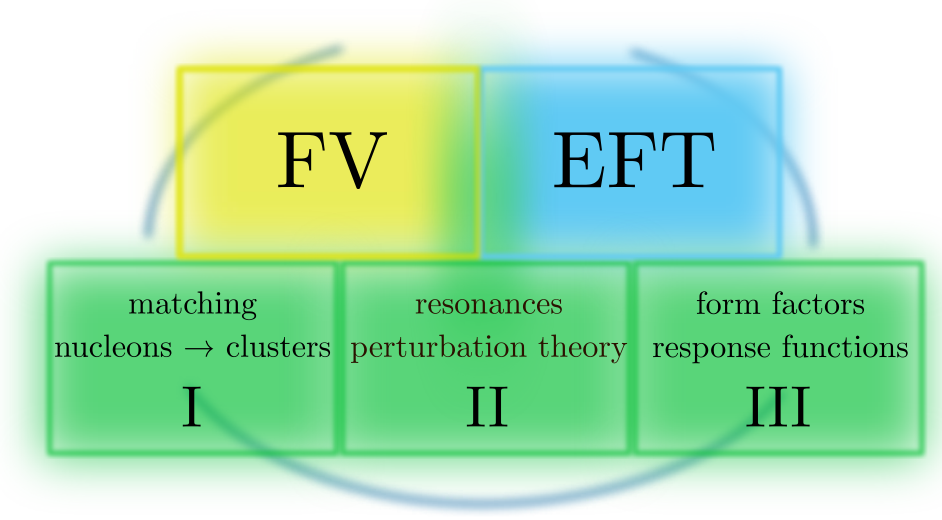
FRIB-TA Annual Meeting

August 9, 2022

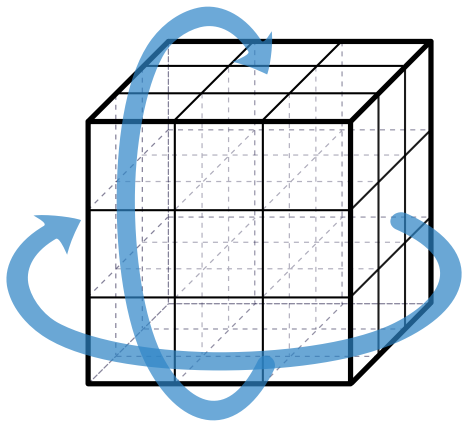
Finite-volume research program

- simulations of quantum systems in **Finite Volume (FV)** can be used to elegantly extract physical properties
- **Effective Field Theory (EFT)** provides a model-independent descriptions of nuclear interactions
- the combination of these two concepts can be used to study a number of interesting questions

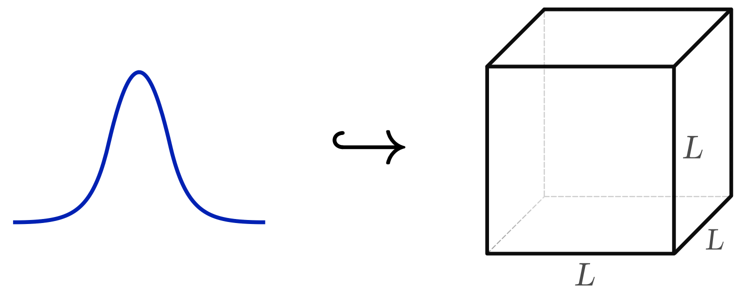


- funded with NSF CAREER grant since last year

Finite periodic boxes



- physical system enclosed in finite volume (box)
- typically used: periodic boundary conditions
- **leads to volume-dependent energies**



Lüscher formalism

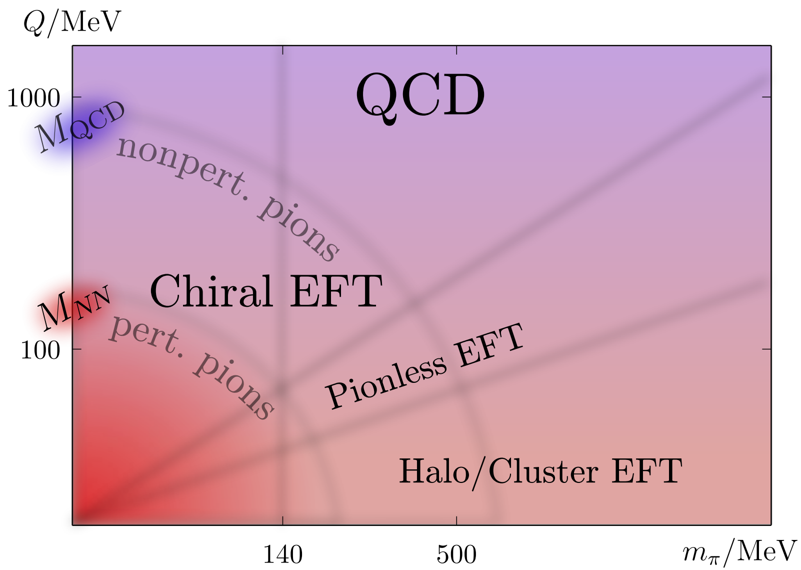
- physical properties encoded in the volume-dependent energy levels
- infinite-volume S-matrix governs discrete finite-volume spectrum
- **finite volume used as theoretical tool**

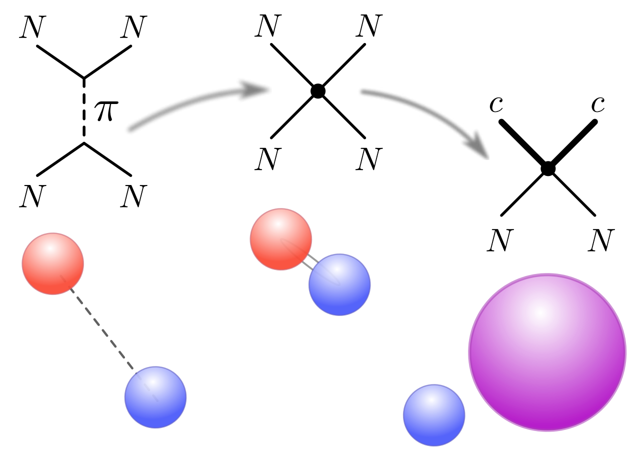
Lüscher, Commun. Math. Phys. **104** 177 (1986); ...

Nuclear effective field theories

- choose **degrees of freedom** approriate to energy scale
- only restricted by **symmetry**, ordered by **power counting**

Hammer, SK, van Kolck, RMP **92** 025004 (2020)



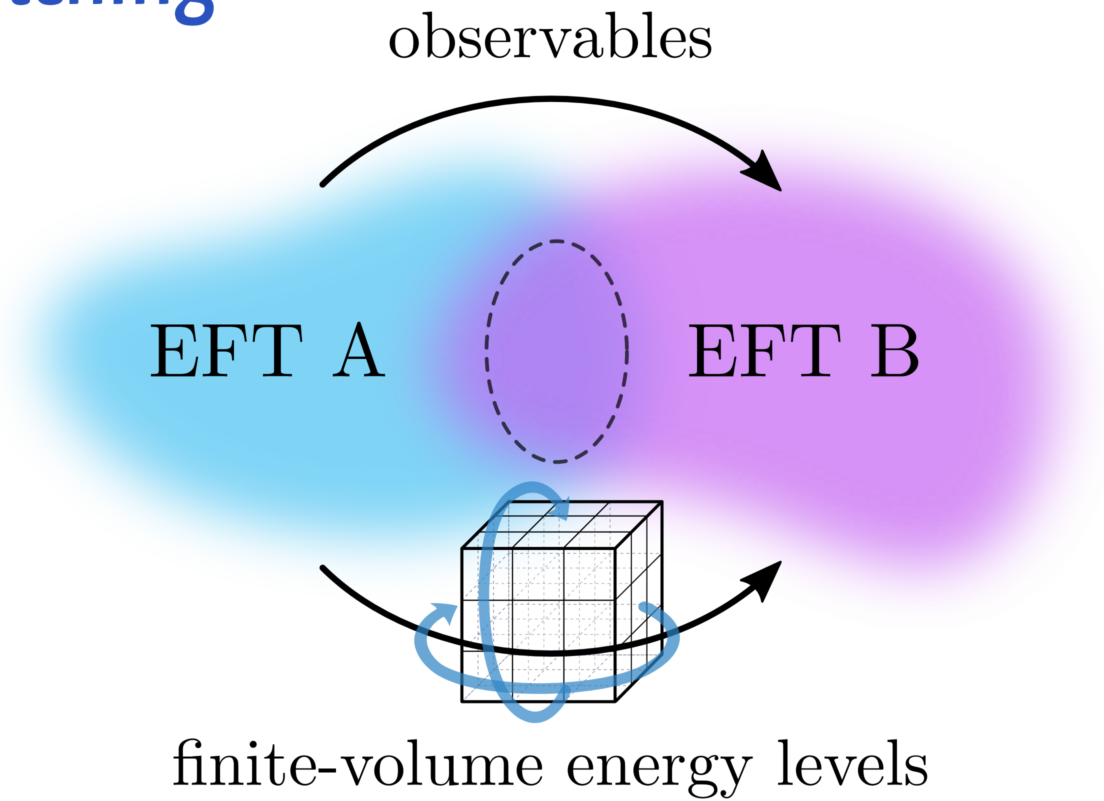


- degrees of freedom here: nucleons (and/or clusters thereof)
- even more effective d.o.f.: rotations, vibrations

Papenbrock, NPA **852** 36 (2011); ...

- most effective theory depends on energy scale and nucleus of interest
- **matching** different EFTs can leverage the reach of ab initio calculations

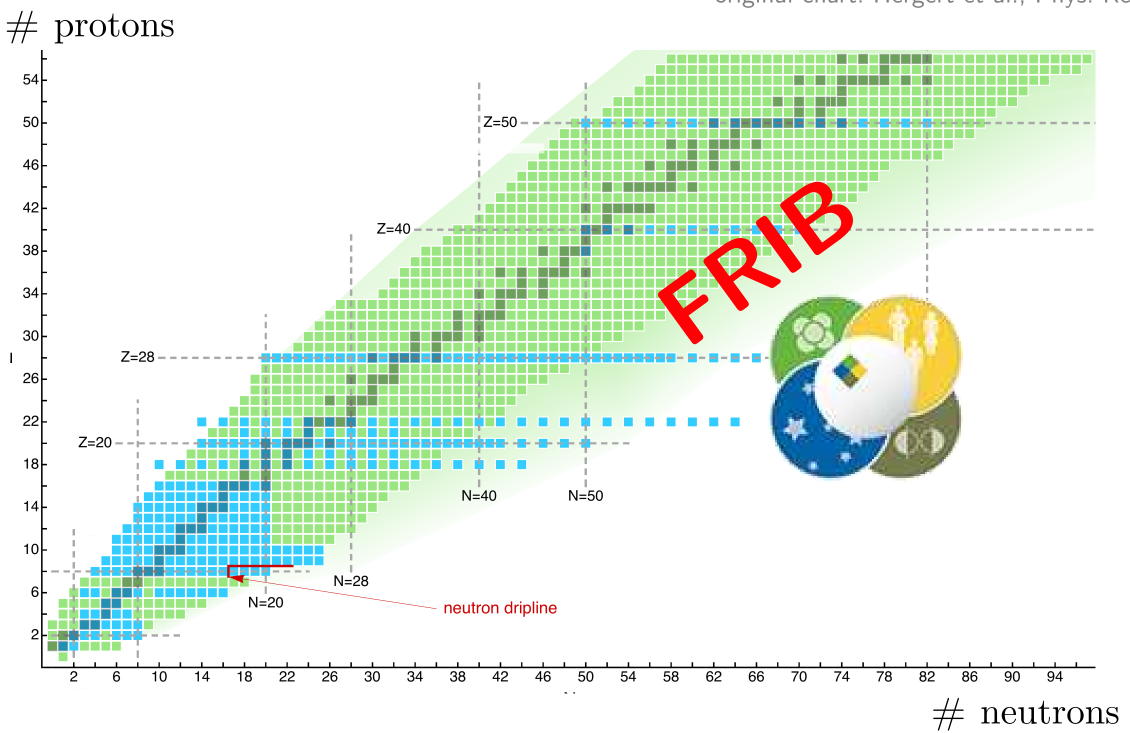
EFT matching



- (E)FTs can be **matched** in their **overlapping regime of applicability**
  - "analytic continuation" of theories
- specifically, the Chiral EFT (Lattice) input can inform Halo/Cluster EFT (FV DVR)

Theory for FRIB physics

original chart: Hergert et al., Phys. Rep. **621** 165 (2016)



- FRIB will measure a host of unknown states near the edge of stability
- theory needs to describe **reactions** and **continuum observables** ($\rightarrow$ resonances!)
- many states in this regime are expected to have a halo/cluster structure

Upcoming Topical Program

Few-Body Clusters in Exotic Nuclei and Their Role in FRIB Experiments

FRIB Theory Alliance

- bridge the gap between theory efforts and needs of the FRIB experimental program
  - in particular FRIB PAC1 experiments involving few-body effects in exotic nuclei
- teams of participants will discuss and write contributions
  - identify important issues
  - **collect ideas how to resolve those**
- organized by: Ch. Hebborn, K. Fossez, L. Platter, SK, starting August 15

Bound-state volume dependence

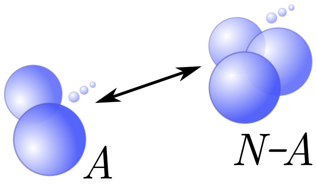
- **finite volume affects the binding energy of states:** $E_B \to E_B(L)$

$$\Delta E_B(L) \sim -|\gamma|^2 \exp\big(-\kappa L\big)/L + \cdots, \; \gamma = \textbf{ANC}$$

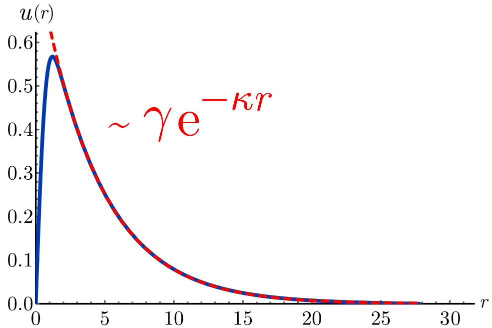
Lüscher, Commun. Math. Phys. **104** 177 (1986); ...SK et al., PRL **107** 112001 (2011); A. Phys. **327**, 1450 (2012)

General N-body result

- volume dependence for arbitrary number of constituents
- nearest breakup threshold determines volume dependence
- **possible to extract ANCs for cluster states**



SK + Lee, PLB **779** 9 (2018)



Low-energy capture reactions

- $p + {}^9\mathrm{Be} \to {}^{10}\mathrm{B} + \gamma$

  Wulf et al., PRC **58** 517 (1998)
- $\alpha + {}^{12}\mathrm{C} \to {}^{16}\mathrm{O}^* + \gamma$
- $\ldots$

  deBoer et al., RMP **89** 035007 (2017), ...
  SK et al., JPG **40** 045106 (2013)

Radius extrapolation

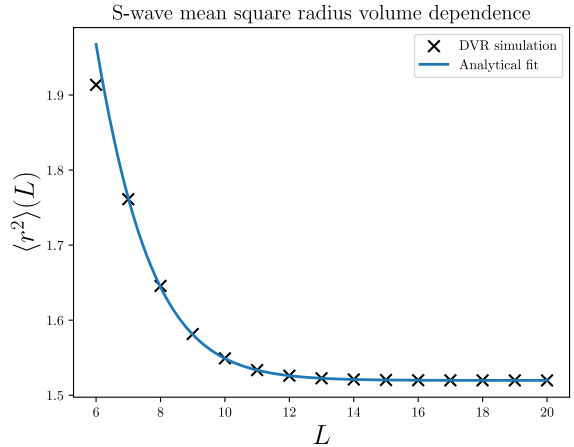
- **all observables carry an inherent volume dependence**
  - specific form depends on the operator of interest
  - in general, not determined by asymptotic properties alone

cf. More et al., PRC **89** 044301 (2014), Odell et al., PRC **93** 044331 (2016)

Example: mean square radii

- volume dependence is exponential in L with polynomial prefactor
- form worked out for S-wave states, higher partial waves being worked on
- fit to numerical data gives excellent agreement
- derivation and analysis by undergraduate student Anderson Taurence

Taurence + König, work in progress



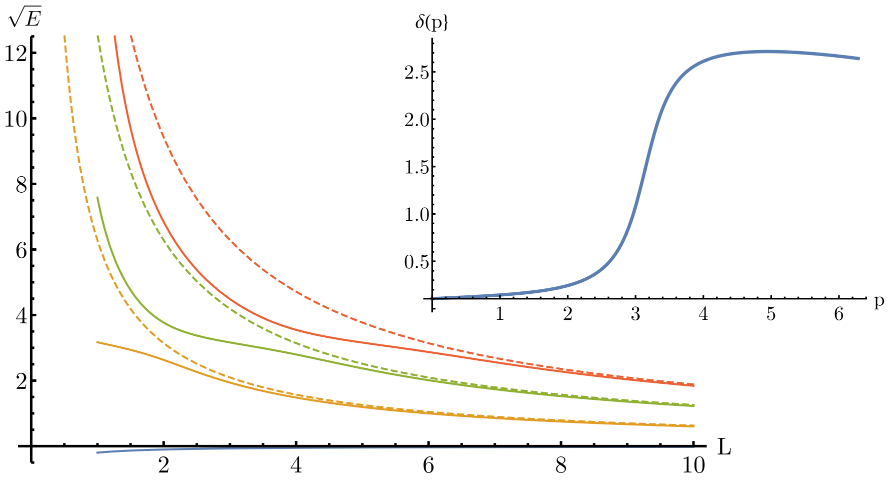
Finite-volume resonance signatures

Lüscher formalism

- finite volume $\rightarrow$ discrete energy levels $\rightarrow p\cot\delta_0(p) = \frac{1}{\pi L} S(E(L))$ $\rightarrow$ phase shift
- **resonance contribution $\leftrightarrow$ avoided level crossing**

Lüscher, NPB **354** 531 (1991); ...

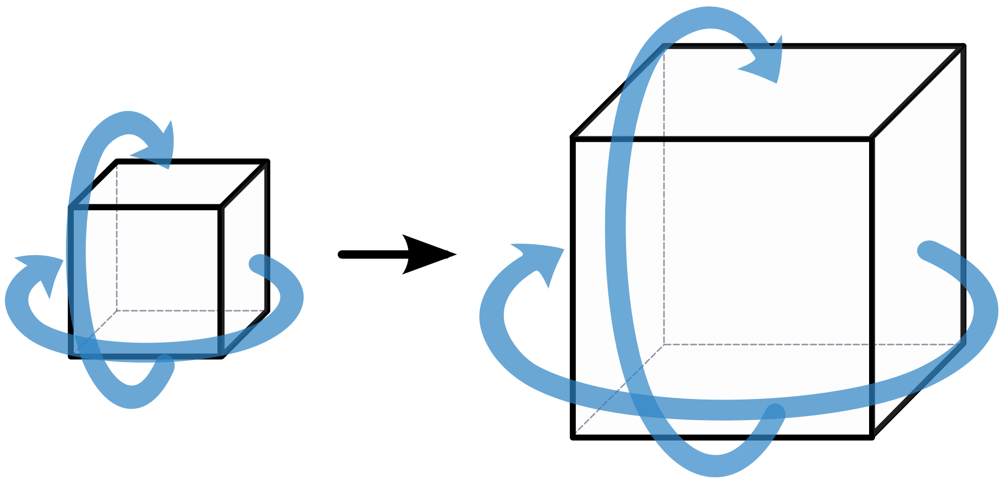
Wiese, NPB (Proc. Suppl.) **9** 609 (1989); ...



- spectrum signature **carries over to few-body systems**
  - ▸ need considerable range of volumes for such studies!

Klos, SK et al., PRC **98** 034004 (2018)

Volume extrapolation

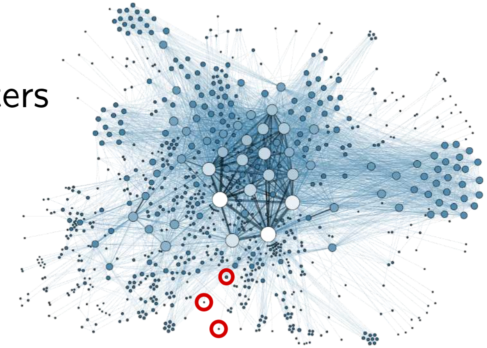


$$L_1 \qquad \to \qquad L_2 \gg L_1$$

Eigenvector continuation

Many physics problems are tremendously difficult...

- huge matrices, possibly too large to store
  - ever more so given the evolution of typical HPC clusters
- most exact methods suffer from exponential scaling
- **interest only in a few (lowest) eigenvalues**



Martin Grandjean, via Wikimedia Commons (CC-AS 3.0)

Introducing eigenvector continuation

D. Lee, TRIUMF Ab Initio Workshop 2018; Frame et al., PRL **121** 032501 (2018)

KDE Oxygen Theme

- novel numerical technique, broadly applicable
  - emulators, perturbation theory, ...

SK et al., PLB **810** 135814 (2020); Demol, ..., SK et al., PRC **101** 041302 (2020); ...

- **amazingly simple in practice**
- special case of "reduced basis method" (RBM)

Bonila et al., arXiv:2203.05282; Melendez et al., arXiv:2203.05528

General idea

Scenario

Frame et al., PRL **121** 032501 (2018)

- consider physical state (eigenvector) in a large space
- **parametric dependence of Hamiltonian $H(c)$ traces only small subspace**

Procedure

- calculate $|\psi(c_i)\rangle$, $i = 1, \ldots N_{\mathrm{EC}}$ in "training" regime
- solve generalized eigenvalue problem $H|\psi\rangle = \lambda N|\psi\rangle$ with
  - $H_{ij} = \langle\psi_i|H(c_{\mathrm{target}})|\psi_j\rangle$
  - $N_{ij} = \langle\psi_i|\psi_j\rangle$

Prerequisite

- smooth dependence of $H(c)$ on c

Result

- construction of highly efficient, tailored variational basis
- enables analytic continuation of $|\psi(c)\rangle$ from $\{c_i\}$ to c_{target}

Finite-volume eigenvector continuation

Naive setup

- consider states $|\psi_{L_i}\rangle$ at volume L_i
- want to use these to extrapolate via EC to target volume L_*
- to that end, we'd consider Hamiltonian and norm matrices like this:

$$
\begin{aligned}
H_{ij} &= \langle\psi_{L_i}|H_{L_*}|\psi_{L_j}\rangle \\
N_{ij} &= \langle\psi_{L_i}|\psi_{L_j}\rangle
\end{aligned}
$$

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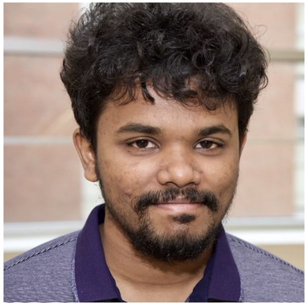
$$N_{ij} = \langle\psi_{L_i}|\psi_{L_j}\rangle$$

However...

All the $|\psi_{L_i}\rangle$ are defined in different Hilbert spaces!

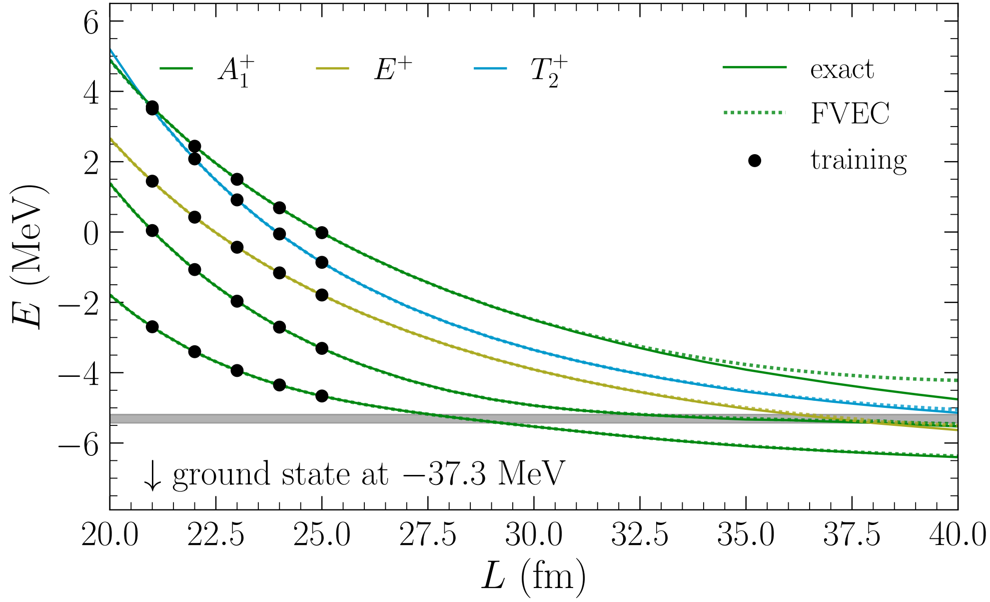
- parametric dependence now not only in the Hamiltonian...
- ...but inherent in the basis
- **can be resolved via periodic matching**
  - union of periodic Hilbert spaces $\mathcal{H} = \bigcup_{L>0} \mathcal{H}_L$
- work together with graduate student Nuwan Yapa

Yapa+König, PRC **106** 014309 (2022)



Three-boson resonance

- three bosons with mass $m = 939.0$ MeV, potential = sum of two Gaussians
- **three-body resonance at**
  - $-5.31 - i0.12$ MeV (Blandon et al., PRA **75** 042508 (2007))
  - $-5.96 - i0.40$ MeV (Fedorov et al., FB Syst. **33** 153 (2003)) (potential S-wave projected!)



- **avoided crossing well reproduced by FVEC calculation**

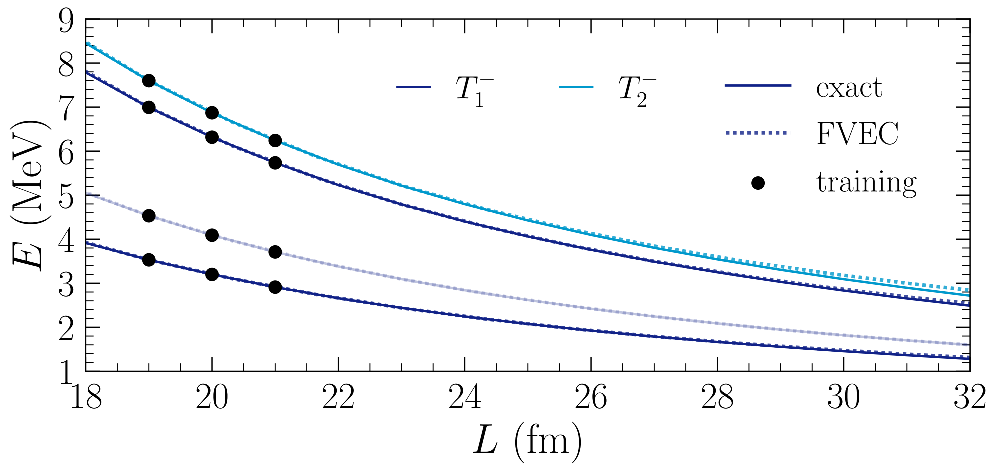
Three neutrons

- consider three neutrons with Pionless EFT leading-order interaction

$$V(q,q') = C\,g(q)g(q') \quad , \quad g(q) = \exp(-q^{2n}/\Lambda^{2n})$$

- separable super-Gaussian form with $n = 2$ and $\Lambda = 250$ MeV
- efficiently implemented within DVR framework

Dietz, SK et al., PRC **105** 064002 (2022)



- total number of training data: $3 \times 8 = 24$ (partly covering cubic group multiplets)
- **four-neutron finite-volume resonance search enabled by FVEC!**

SK et al., work in progress

Outlook

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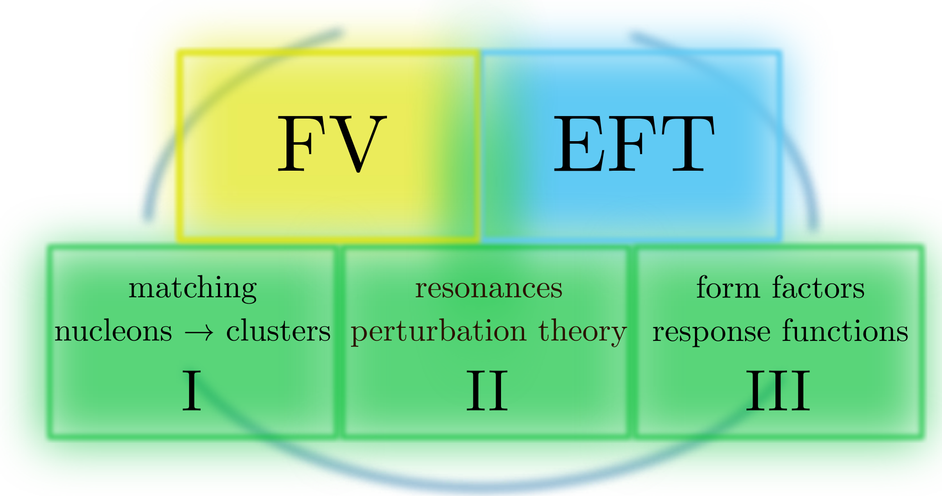

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Concrete current projects

1. **EFT for α-cluster states**
    - build Halo EFT based on large α-α scattering length
    - properly include Coulomb interaction
2. **Charged particles in finite volume**
    - important to determine ANCs for relevant systems
    - worked on by graduate student Hang Yu
3. **Resonance eigenvector continuation**
    - extrapolate between bound states and resonances
    - works well in FV, studied now in momentum space
    - worked on by graduate student Nuwan Yapa
4. **Radius volume extrapolation**
    - shown earliear in this talk
    - worked on by undergraduate student Anderson Taurence

Experimental wishlist

States

- **neutron-rich isotopes of elements with doubly magic ground states**
  - $^{12}C + X \cdot n$, $^{16}O + X \cdot n$, ...
  - good candiates for hosting unknown halo configurations
- **proton halo nuclei**
  - exposing the delicate interplay of nuclear attraction and Coulomb repulsion
- **genuine few-body resonance states**
  - no decay into two-body final state

Observables

- **binding energies of stable halo states**
- **for resonances, identification of quantum numbers, energies, and lifetimes**
- **radii, form factors, and response functions**
  - to determine electromagnetic properties

Thanks...

...to my students and collaborators...

- **N. Yapa, H. Yu, A. Andis, A. Taurence (NCSU)**
- D. Lee (FRIB/MSU), K. Fossez (ANL)
- S. Dietz, H.-W. Hammer, A. Schwenk (TU Darmstadt)
- P. Klos, J. Lynn
- ...

...for support and funding...

FRIB Theory Alliance

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...and to you, for your attention!